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Valuation and investment of generation assets

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NOTE TO USERS

Valuation and investment of generation assets

by

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For the Major Program

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Abstract

The re-regulation of electric power industry around the world has raised many new challenges for all stakeholders. This research is to valuate generation assets within re-regulated electricity markets, both in short-term and long-term. The focus is to valuate operation flexibility under market uncertainties from the viewpoint of a Generation Company (GENCO).

This research proposes to model the movements of electricity markets with Hidden Markov Model (HMM) driven by underlying market forces. An electricity market is modeled as a dynamic system evolving over time according to Markov processes. At any time interval, the electricity market can be in one state and transit to another state in the next time interval. The true market states are hidden from a market participant behind the incomplete observation. The observations, such as market-clearing price and quantity, are modeled to follow multiple probabilistic distributions.

This research proposes to further decompose the market forces into physical and economic drivers if a specific electricity market employs Location Marginal Price (LMP) mechanism. The physical drivers include transmission network topology and generation technology. The economic drivers include fuel prices, demand uncertainties, and profit maximization of market participants with incomplete information. The decomposition captures the strengths of engineering-based production cost approach and mark-to-market stochastic approach.

This research valuates generation assets with real option analysis. The value of generation assets is maximized based on the Hidden Markov Model (HMM) and newest observation of electricity markets. Such an optimization problem is formulated as Partially Oberserable Markov Decision Problem (POMDP). The solution of a POMDP provides a GENCO both the optimal operating policy and values of generation assets. The value of perfect and imperfect information is also identified.

Investment in generation assets is also analyzed with real option. This research incorporates fuzzy sets and numbers to capture the fuzziness and possibilities of long-term electricity markets movements. Fuzzy sets and numbers provide the modeler flexibilities to

incorporate subjective judgments when rigorous approaches are not feasible. The real call options, capturing the investment value of generation assets, are formulated as Markov Decision Process (MDP) and solved with fuzzy linear programming.

Chapter 1: Introduction

This chapter gives an introduction to the re-regulation of the electric power industry. The objective of this work is outlined in section 1.2. Section 1.3 provides a market framework assumed for this work. The accomplished work and contributions of this research are given in Section 1.4. This chapter concludes with an introduction to the organization of this document.

1.1 Electric Power Industry Re-regulation

The economic incentives to pursue cheap and reliable electric power supply have led the electric power industry from un-regulated to regulation then re-regulation. Appendix A-1 summarizes the evolution of the U.S. electric power industry and defines the background of this document. More details on the evolution of U.S. electric power industry could be found in [1].

The electric power industry re-structuring has proposed, investigated, and implemented a variety of electricity market models. Those models differ in their market structures and market architectures. Market structure is defined as properties closely tied to technology and ownership. Market architecture refers to a set of sub-markets and the linkages between them [1].

The notion of market structure was developed initially as part of the "structure-conduct-performance" paradigm of industrial organization in the early 1950s [3]. The classic market structure focuses on the ownership of production capacity. One of the most popular indexes is Hirschmann Herfindahl Index (HHI), which is defined as in Equation 1. A regulated utility is given the franchise to operate in a regional electricity market, which leads to an HHI = 10000.

$$HHI = \sum_{i=1}^{4} X_i^2, \ \forall i, X_i \geq X_{i+1} \geq 0$$
$$X_i\% = Market\ Share\ of\ Producer\ i,$$

Equation 1: Definition of Hirschmann Herfindahl Index

Regulatory agencies re-constructs electric power industry by breaking vertically integrated electric utilities into horizontally independent entities including Generation Company (GENCO), Transmission Company (TRANSCO), Distribution Company (DISCO), Electric Service Company (ESCO), and Electricity Management Company (EMCO) among other possibilities. The institutional decoupling of electric power generation, transmission, and distribution lays down the economic foundation of electricity markets. The physical foundation of electricity markets, open and equal access to transmission and distribution networks, is provided by Independent System Operators (ISO)/Regional Transmission Organizations (RTO). Both the physical side and financial side of an electricity market are under the authority of regulatory agencies. Figure 1 shows a conceptual market structure.

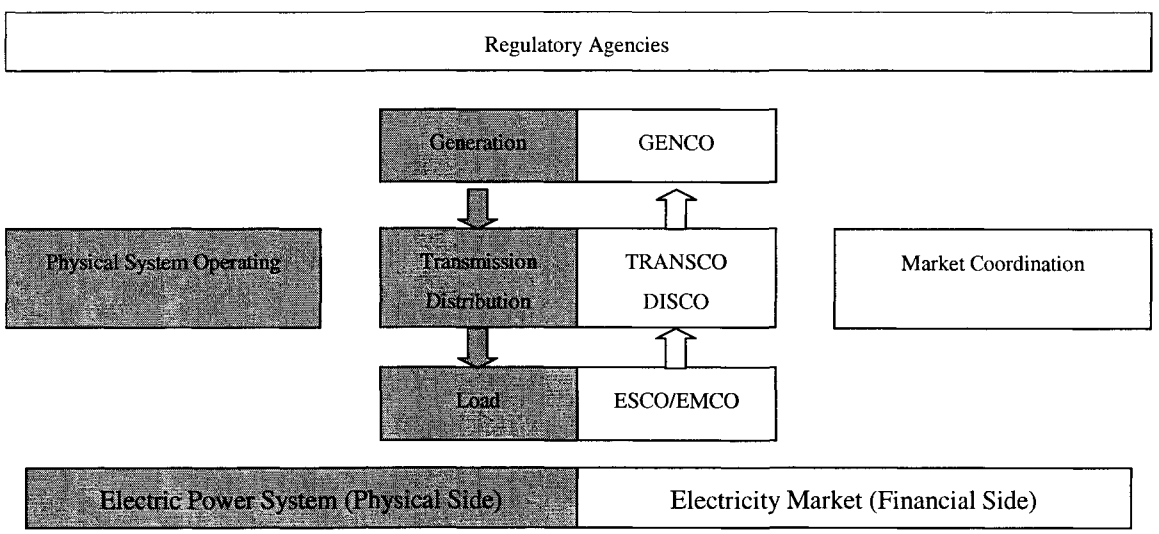


Figure 1: Market Structure of Electricity Market

Most restructuring models share the decoupling of vertically integrated utilities. However, significant differences in market structures have been observed. The differences

mainly concern on the assignments of physical system operating and market coordination. Some electricity markets institutionalize business intermediaries, who deal directly with transmission service providers on behalf of GENCOs and ESCOs/EMCOs. Such intermediaries include centralized, wholesale power pools or Power Exchanges (PX), as well as aggregators of bilateral energy service contracts, generically referred to here as Scheduling Coordinators (SCs). Not all of these entities need to be present in any specific restructuring model. In some cases the centralized energy market (PX) does not exist, and in other cases it is merged with the ISO. The bilateral market represented by the SCs may or may not be provided, depending on the structure adopted. Figure 2 shows the structural difference between existing and evolving electricity markets, where adjoining boundaries indicate that the services are provided by a single entity. A brief comparison of some exiting electricity market frameworks can be found in Appendix A-2. The timeline for the coordination work carried out by California ISO and SCs are illustrated in Appendix A-3.

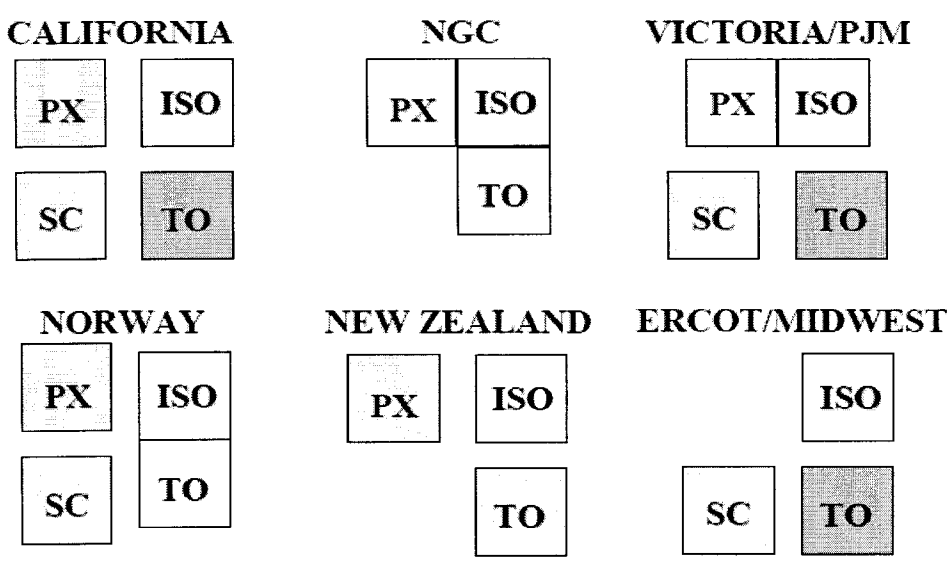


Figure 2: Difference of Market Structure

The technology part of electricity market structure has also seen significant developments, both in electricity generation and transmission. The advances of electric power generation technology have changed the generation stack. Natural gas has become a

significant source of generating electric power for high efficiency of combined cycle combustion unit and low emission cost. Renewable energy has seen more and more commercial installments, especially wind energy. Table 1 illustrates the electricity generation composition of several regional electricity markets in North America [4]. It is shown that the generation composition could be significantly different between electricity markets. The development of control and communication technology provides essential supports for offering open and equal access to an integrated electric power system.

Table 1: Generation Composition of North American Electricity Markets

	PJM	NERTO	NYISO	ERCOT	CA
Gas	31%	32%	90%	75%	46%
Oil	21%	28%	74%	38%	1%
Coal	37%	9%	0%	21%	0%
Nuclear	21%	16%	0%	7%	9%
Hydro	5%	12%	1%	1%	23%
Others	1%	2%	0%	0%	21%

Some generator could switch fuels, thus the sums of percentages do not necessarily equal to 100%.

Market structure defines the players of an electricity market and their competition positions while market architecture defines how the entities interact and exchange both commodities and information. The market architecture design must consider the market structure in which it is embedded, which may inhibit the proper function of some designs.

Market architecture defines a set of sub-markets and the linkages between them. The linkages between sub-markets may be implicit price linkages bound by arbitrage or explicit rules linking rights purchased in one market to activity in another. The sub-markets of an electricity market could be defined by many criteria, including:

- Commodity traded

The main physically deliverable commodities traded in electricity markets are electric energy and ancillary services. In some market architecture, electric energy and ancillary services are bundled together while other markets trade them separately. The discussion on

whether electric energy and ancillary services should be fully un-bundled is still under discussion and more details can be found [5][6]. This research models electric energy and ancillary services to be unbundled.

The linkages between electric energy and ancillary services are two-folded. On the complementary side, ancillary services are necessary for the transfer and delivery of electric energy. On the competing side, the production of electric energy and ancillary services consumes the same resource, infinite generation capacity.

- Contracts traded

There are four major kinds of contracts traded on electricity markets, spot, forward, future and options. Forward contracts are normally traded for physical delivery, and it allows the scheduling of both generation facilities and transmission networks operation. The day-ahead electricity market is an exchange forward market. Spot contracts, also known as real time markets, are used to allow re-scheduling and forecast errors. Future contracts are more often used as hedging instruments, while physical delivery is also possible. Although NYSE, CBOT and other exchange operated trading of future on electricity, those markets were suspended for lack of liquidity. Options are derivatives written on underlying assets. More information on spot, forward, future and options could be found in [7].

- Trading mechanism

A few trading mechanisms are in use now, and they include bilateral contracts, electric power pool, and electricity exchange. Most electricity markets employ multiple trading mechanisms. Bilateral contracts, also referred to Over The Counter (OTC), allow more flexibility, but limit the information sharing among all market players and price discovering. Also, bilateral contracts introduce default risk to both sides of a specific transaction. Forward contracts are often traded as OTC. An electricity pool does not allow a GENCO to make decision on unit commitment and dispatch, but solve the unit commitment and economic dispatch based on the "virtual production curves" submitted by GENCOs. A closely connected and monitored electricity pool is often the first stage of re-regulating

electric power industry and building electricity markets. An electricity exchange eliminates credit risk, facilitates information sharing, and increases the market liquidity. GENCOs are given more flexibility on the operation of generation assets.

1.2 Problem Statement

Valuation of generation assets is a basic yet critic problem of electric power industry. The re-regulation of electric power industry brings two challenges to the valuation of generation assets, the uncertainties associated with electricity markets and the operating flexibilities awarded.

The electricity markets demonstrate significant uncertainties from the viewpoint of valuating generation assets. Electric energy and ancillary services are traded according to their time-varying and location dependent values [8]. A few factors, including:

- Non-existence of economic storage on electric power
- Instantaneously balanced electric power supply and demand
- Limited capability to transfer electric power
- Limited capability to adjust demand in short notice

contribute to significant volatilities observed on electricity markets. However, electricity markets also demonstrate seasonal, weekly, and daily patterns. The modeling of both the patterns and uncertainties of electricity markets is the first problem to be addressed for the purpose of valuating generation assets.

Electricity markets also award GENCOs with flexibilities on operating generation assets. A profit-maximizing GENCO has full control on its generation assets, including self unit-commitment and dispatch. However, the flexibilities are subject to complex constraints. The most important generation assets' operating constraints are time constraints including start up time, shut down time, minimum on time, minimum down time, ramp up rate and ramp down rate. Startup and shut down time state that a generator cannot be started up or shut down immediately, which means the decision to change the operating status must be made before the prices on electricity and fuel are known. Minimum on and down time states a generator must remain on or off once being started up or shut down, which means operating

flexibilities are not always available and losses are possible. Ramp on and ramp down rate limits how fast a generator's output could change once on. The impacts of ramp up/down rates are two folded. First, it means that the full generation capacity is not always available. Second, it introduces lags between decision made to change the operating status of generation assets and the intended results. The modeling of such operating flexibilities is the second problem to be addressed for the purpose of valuating generation assets.

This research is to valuate generation assets within re-regulated electricity markets, both in short-term and long-term. The focus is to valuate operation flexibility within uncertain re-regulated electricity markets. This research includes two closely related parts:

- New models for electricity markets and generation assets: An open framework to incorporate engineering insights and market signals
- Market based valuation and decision-making tools: Short-term valuation and Long-term planning and investment

The modeling of re-regulated electricity markets aims to efficiently use of available information while still keep tractability and practicality. The valuation and decision-making tools aim to incorporate the operation flexibility while respecting physical constraints of generation assets.

1.3 Electricity Market Model Assumed for this Work

The electricity market framework design has been under extensive research, and attention has been paid to adoption to local needs such as generation technology, load pattern. This research assumes a market framework proposed by Gerald B. Sheblé [9], and it is illustrated in Figure 3. It is a short-term framework, where mid-term and long-term contracts are not presented. This market framework is the minimum set to illustrate this work while still maintaining tractability in a dissertation. The market framework includes forward and real time markets for electric energy and ancillary services. All markets trade on exchange where a double-auction mechanism is implemented. Details on modeling electricity markets in short-term, mid-term and long-term could be found in chapter three.

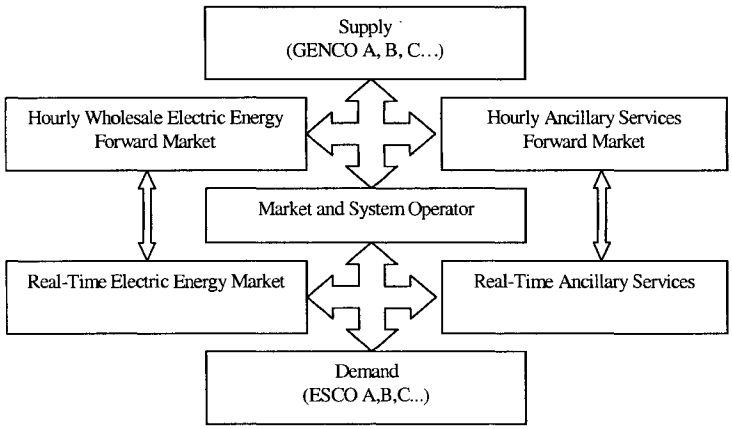


Figure 3: Minimum Set of Electricity Markets

1.4 Achievements and Contribution of this Work

The contribution of this work includes the modeling of electricity markets and the valuation of generation assets. A list of the publications based on this work is given in Appendix A-4.

This research proposes to model the movements of electricity markets as partially oberserable Markov processes driven by underlying heterogeneous forces. An electricity market is modeled as a dynamic system evolving over time according to Markov processes. At any time interval, the electricity market can be in one state and transit to another state in the next time interval. This work models the states of an electricity market as partially observable, while each state has incomplete observations such as market-clearing price and quantity. The true market states are hidden from a market participant behind the incomplete observation.

One application of HMM is of a more general approach and focuses on capturing the interaction of demand and supply forces on electricity markets. This approach extends and enhances the approach of regime switching. A fully constructured regime/state space is proposed, while the dynamics of the switching/transiting is modeled to follow Markov processes. This approach captures the unique features of electric power as a special commodity. This model is shown to be able to captures the observed clustering of

volatilities, price spikes and other phenomenon on electricity markets. Although this approach is a Mark-To-Market (MTM) approach, HMM could also apply to incorporate engineering insights.

Another application of HMM is the modeling of Location Marginal Price (LMP). The movement of LMP is decomposed into physical and economic drivers. The physical drivers include transmission network topology and generation technology. The economic drivers include fuel prices, demand uncertainties, and profit maximization of market participants with incomplete information. The electricity market and power system are modeled to transit between different system states according to Markov chains defined by physical drivers. System-states-depending random processes capture the impacts of economics drivers. Combined together, this new approach models LMP to be generated from multiple random processes. This model combines the strengths of both fundamental economic modeling and mark-to-market stochastic modeling. The physical driver captures the strengths of engineering-based production cost modeling approach while the economic drivers captures the strength of mark-to-market approach. The decomposition provides a modeler the capability and tools for structural modeling of LMP. The key market drivers could be identified and be appropriately modeled to capture the heterogeneous nature of electricity.

HMM could also be extended to short/mid/long-term modeling of electricity markets. In short-term, the market movements are modeled as market state transition. Mid-term market evolvements are captured by Model parameter changes. Long-term market evolvements are modeled as Model structure changes. Figure 4 illustrates a market movement path by linking multiple Markov models.

This work proposes to valuate generation assets with real option analysis. Real option analysis provides GENCOs with a new methodology to fully valuate the operating flexibilities, which can be flexibilities inherent in the nature of generation assets or flexibilities traded on markets. A Generation Company (GENCO) is modeled to maximize expected profit based on the Hidden Markov Model (HMM) of electricity markets and newest observation on the electricity markets. This profit maximization process is modeled as a Partially Observable Markov Decision Problem (POMDP). A POMDP is identical to a

real option problem where uncertainties are structured to evolve as Markov processes. A formulation of POMDP is shown in Figure 5. The option value of generation assets could be identified and optimized. Values of perfect and imperfect information could also be provided by the differences under different market observability.

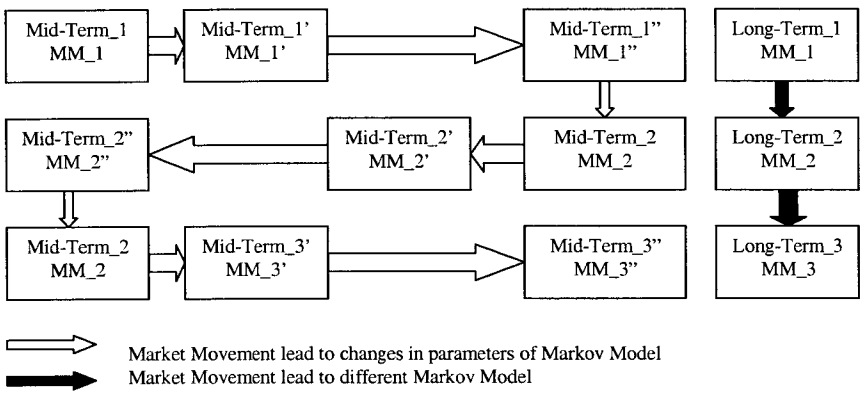


Figure 4: Markov Model for Short/Mid/Long-Term Modeling of Electricity Markets

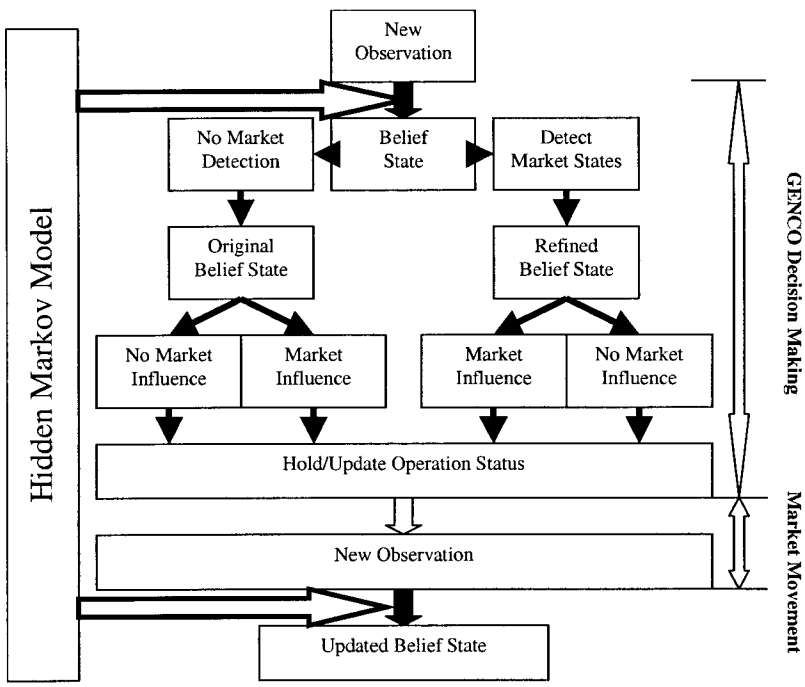


Figure 5: POMDP for Generation Assets Valuation and Operation as Real Option Analysis

This work proposes to capture the uncertainties and fuzziness of long-term market forces by fuzzy numbers. Valuation of generation assets in mid-/long term is modeled as fuzzy real call options on synthetic spreads. The fuzzy real call options are formulated as Markov Decision Processes (MDP). A MDP with fuzzy features is constructed as a fuzzy linear programming problem. A fuzzy linear programming problem is transformed into a set of regular linear programming problem, and then solved.

1.5 Organization of Dissertation

This dissertation is organized into five parts. Chapter one introduces the re-regulation of electric power industry and this work. Chapter two reviews the past research done on valuating generation assets both under regulation and re-regulation. Chapter three focuses on the modeling of electricity markets. Chapter four discusses the valuation of generation assets. Chapter five concludes this dissertation. More background information could be found in the Appendix.

Chapter 2: Literature Review

This chapter reviews researches done on valuation of generation assets both under regulation and re-regulation. Section 2.1 reviews the past practice of electric utilities on operating and investing in generation assets. Section 2.2 and 2.3 reviews the latest research on modeling electricity markets and valuating generation assets. Section 2.4 discusses the long-term planning and investment of generation assets. Section 2.5 concludes this chapter and outlines the missing parts on both modeling electricity markets and valuating generation assets.

2.1 Generation Assets Operation under Regulation

Before Re-regulation, an electric utility was granted both the franchise to operate as a monopoly and the obligation to serve all demand rising in its own franchised geographic area. Return on investment was guaranteed and fixed at a rate, the rate of service. Regulatory agents reviewed the rate of service periodically to keep the return rate of the franchised utility at a reasonable level. The rate of service remained fixed during two reviews. Under unfavorable situations, electric utilities could appeal for the increase of rate of service, thus keep the return rate at a reasonable level. It is often the case that an electric utility being asked by regulatory agents to pass its saving of cost to customers. Under cost-based regulation, the values of generation assets were known with certainty.

In short-term, profit-maximization was equivalent with cost-minimization for an electric utility with a given fixed rate of service. Given the obligation to meet all demand rising in its franchised area, an electric utility minimizes its production cost by "unit commitment" and "economic dispatch". The objective of "unit commitment" and "economic dispatch" is to searching for the optimal set of online generation units and production levels. Both unit commitment and economic dispatch assume a set of installed generation assets. Unit commitment concerns optimizing the set of online generation units, namely which

generation unit to turn on/off and when. Economic dispatch concerns optimizing the production levels for each online generation unit, namely how much to produce for an already online generation unit.

There are physical constraints imposing on unit commitment and economic dispatch. The constraints include

- Time Constraints:
  - Start up time, time required to turn an offline generation unit on
  - Shut down time, time required to turn an online generator off
  - Minimum on time, time required to keep an online generation on once turned on
  - Minimum down time, time required to keep an offline generation off once shut down
  - Ramp up/down rate, speed to adjust the production level for an online generation unit
- Capacity Constraints:
  - Minimum output, minimum production level for an online generation unit
  - Maximum output, maximum production level of an online generation unit
  - Unstable output interval, production level intervals where an online generation unit could not operate stably
- Transmission and Security Constraints:
  - Constraints imposed by the stable operation of an electric power system, intervening the economic merit based generation units commitment and dispatching

Under regulation, both of unit commitment and economic dispatch were performed centrally within utilities.

In long-term, the incentives to minimize production cost came from the regulatory agents who tried to maximize social welfare and required electric utilities to minimize cost as a requirement for granting franchise. An electric utility minimizes its production cost of meeting forecasted electricity demand by planning on generation addition and transmission

expansion. The planning problem is extremely hard to solve for the complicated interaction between transmission networks and generation capacities.

Transmission network and generation capacities are substitutes to some extend. A strong transmission network reduces the total generation capacity needed while generation capacity near load pocket removes the needs for a strong transmission network. Transmission network and generation capacities are also complimentary to some extend. The availability of hydroelectric resources helps to justify the needs to expand transmission network while a strong transmission network helps to realize the benefits of adding more efficient capital-intense generation assets located away from load centers. The dynamics of transmission network expansion and generation assets are tightly correlated.

A global approach is desirable to optimize the planning of electric power systems. However, the complexity and high non-linearity of electric power systems planning made a global approach impractical. Before re-regulation, electric utilities decoupled the planning problem into two sub-problems, addition of generation and expansion of transmission networks. One sub-problem was solved by holding the other constant, and this iteration proceeded until a given criteria of convergence is met, as shown in Figure 6.

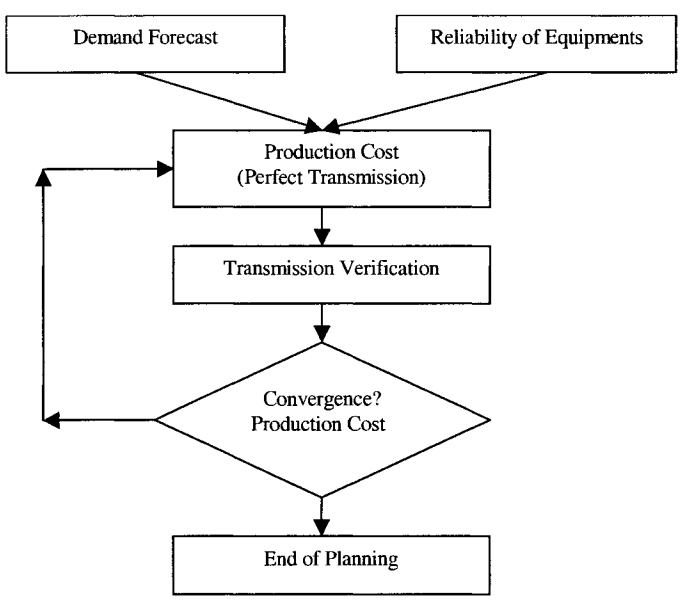


Figure 6: Flow Chart of Power System Planning Practice before Deregulation

For the generation addition sub-problem, a planner forecasts the future load and makes decisions on when to add new generation and the physical characteristics of new generation units assuming perfect transmission networks. The traditional financial tool, discounted cash flow (DCF), was used for selecting optimal generation addition plan. DCF approach was appropriate for there was no risk associated with investments in generation assets. After planning, new generation addition projects were passed from engineering department to finance department to be financed. Such a cost-based practice is in fact an "Engineering leads financing" approach. In fact, the cost-based regulation mechanism provides no economic incentives for generation addition planning. Equation 2 shows the WIEN Automatic System Planning Package (WASP-III) model for generation planning [9]. Under regulation, the transmission networks were also built on the cost-based principle to support the reliable operation of an electric power system. This study focuses on the electricity generation segment, while more discussion on the expansion of transmission network can be found [10][11].

$$min\ PVC_j = \sum_{t=1}^{T} [\bar{I}_{jt} - \bar{S}_{jt} + \bar{F}_{jt} + \bar{M}_{jt} + \bar{O}_{jt}]$$

Subject to :

Energy Balance

$$(1+a_t)PeakLoad_t * AvailableGenerationCapacity \leq (1+b_t)\ PeakLoad_t$$

$$(AvailableGenerationCapacity - PeakLoad)_t = \underset{t=1}{\overset{T}{Min}}(AvailableGenerationCapacity - PeakLoad)$$

Reliability Constraints

$$LOLP(Loss\ of\ Load\ \ \mathrm{Pr}obabilty) \leq C$$

Where PVC = Present valud of the total cost for a given scheme j

I = Investment Cost

S = Converted remnant value of investment

F = Fuel Cost

M = Cost of operation and Maintenance

O = Outage Cost

a_t *and* b_t *: Heuristic average weight factor*

Equation 2: WASP Generation Planning Problem

2.2 Modeling of Re-regulated Electricity Markets

On re-regulated electricity markets, the demand and supply forces discover the price of electricity. Often, the prices of electricity diverge from the production cost of electric energy but reflect the time-varying and location-dependent values of electric energy. The modeling of electricity prices is a fundamental yet critical problem confronting all GENCOs. There are two approaches for modeling the dynamics of electricity prices: Mark-To-Market (MTM) time series approach and engineering-based production cost modeling approach.

2.2.1 Mark-To-Market Timer Series Approach

The MTM time series approach models electricity prices as autonomous time series, which could be forecasted by investigating only the historical prices. The time series approach for modeling electricity prices is borrowed from the modeling approaches for stock markets and other financial markets. It implicitly assumes weak-form market efficiency and stationary markets.

Geometric Brownian Motion (GBM) with mean-reversion and seasonality means is the foundation of most models for commodities price modeling as shown in Equation 3. It provides a reasonable starting point for modeling electricity markets. The cyclical nature of electricity price movement is captured by the seasonal pattern, while the commitment and dispatch behavior of GENCOs are represented with mean-reversion. GBM with its modifications have been applied to valuate both transmission and generation assets [12][13][14][15][16][17].

$$d(\ln P_t^E) = m^E(\ln P_t^E - m_t^E)dt + s^E dW_t^E$$

$$\begin{aligned} \text{Where} \quad & m^E : \text{mean-reversion coefficient} \\ & s^E : \text{volatility} \\ & m_t^E : \text{seasonal pattern} \\ & W_t^E : \text{Weiner Process} \end{aligned}$$

Equation 3: Mean Reversion GBM

Many assumptions employed in modeling the behaviors of stock markets should be scrutinized before being applied to electricity markets. The mean-reversion feature of electricity price movements has been tested, and it was found that electricity prices short-term movement does not show mean-reversion [4]. GBM model also assumes that the prices for electricity follow lognormal distributions. Such an assumption is not valid for electricity markets. It is shown that electricity prices follows distributions with fat tails compared to normal distribution. Other features observed on electricity prices movements include extreme volatilities (100%-500% and higher), clustered price spikes, stochastic volatilities and others.

Improvements on the GBM include regime switching [18][19], jump-diffusion processes, General Autoregressive Conditional Heteroskedasticity (GARCH) and others [20]. Regime switching states that there are unobserved market regimes following stochastic processes underlying the observed price movements. Often, electricity markets are categorized into stable regime with less volatility and unstable regimes with extreme volatility. For each regime, an individual econometric model is proposed and fitted with historical data. Jump-diffusion assumes that the movements of electricity prices are not necessarily continuous. The movements of electricity prices are modeled to include: diffusion and jumps. Often, Poisson distributions are employed to model the frequency of price jumps and the sizes of jumps. It is also proposed to model volatility to be dependent on other factors such as time, price levels among others. General Autoregressive Conditional Heteroskedasticity (GARCH) are one implementation of volatility surfaces. GARCH aims to capture the clustering of volatility on electricity markets.

Although complex models are more capable to capture the observed features of electricity price movements, complicated models also find difficulties in parameters estimation and suffer over-fitting problems in econometric sense.

The over-fitting problem is mainly due to the lack of historical data and non-stationary nature of electricity markets. The electricity markets posses short history, and keep evolving. Both the regulators and market participants are in the transition of restructuring. Their behaviors are experiencing significant evolvements. This not only limits

the availability of market historical data, but also raises the problem of choosing appropriate time windows for parameter estimation.

Another problem associated with stationary electricity market assumption is the unstable correlation between electricity prices at different transmission buses, most importantly, the cross-correlation between electricity prices and prices on fuels. The investigation of such correlation is of importance for valuing generation and transmission assets.

Another limiting and questionable assumption is that all the samples observed are drawn from the same distribution. This implies stationary and integrated markets driven by the same physical and economic drivers. The limited transfer capability of transmission network segments electricity markets into isolated zones therefore renders electric energy within different zones as heterogeneous commodities.

The time series approach implicitly assumes weak-form market efficiency for electricity markets, and only employs historical market data. The modeling of imperfect electricity markets driven by heterogeneous physical and economic drivers requires a detailed model for the fundamental drivers. The incorporation of fundamental drivers also addresses the over-fitting problem, unstable correlation and other problems faced by mark-to-market approach.

2.2.2 Engineering-Base Production Cost Modeling Approach

The electricity price movement could also be modeled using fundamental engineering-based approaches such as production cost modeling. Production cost modeling approximates the Location Marginal Price (LMP) algorithms used by ISO/RTOs as shown in Figure 7. This approach requires tremendous input data including detailed presentation of transmission networks, all generation units, demand on electric power, costs and bidding behaviors of all GENCOs among others.

The demand for electricity is forecasted utilizing techniques traditional applied by utilities. With forecasted electric load demand and fuel cost, the marginal generation units are identified based on unit commitment and economic dispatch. Power flow and

contingency analysis identify the binding transmission constraints. The most significant feature of production cost modeling is the incorporation of the detailed representation of transmission systems. Inclusion of transmission systems helps to identify the congested transmission paths, therefore the contribution factors to calculate LMP [21].

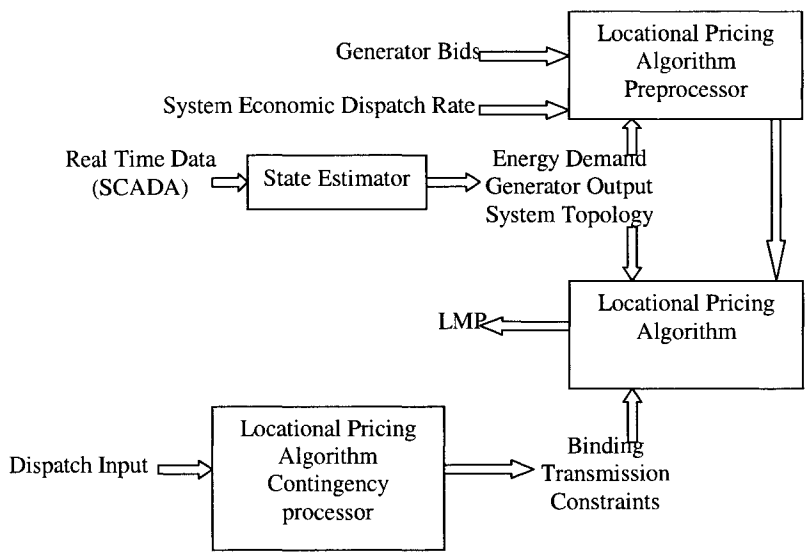


Figure 7: LMP Mechanism

Production cost modeling captures the physical drivers of electricity markets and the cost structures of electricity markets. The incorporation of physical drivers helps explaining the correlation between LMPs, prices of fuels, and demand levels. However, the data and computation burden of simulating electric power systems limits the capability of production cost modeling to capture the uncertainties of market drivers. This severely limits the potential to incorporate production cost modeling into existing valuation and risk management systems.

Another limiting assumption of production cost modeling is that GENCOs bid at their variable cost, including the fuel cost, O&M cost and other variable costs. The profit-maximizing operation of a GENCO is based on its expectation for markets. The GENCOs' expectations for markets are based on incomplete information about the markets, which often lead not to bid at the its producing marginal cost. Detailed modeling of GENCOs' competition behavior is prohibitive expensive, if possible.

Although production cost modeling incorporates public information such as fuel prices, demand forecast and parameters for generators and transmission systems, it fails to utilize historical market data. The ignorance of historical market data severely limits the application of production cost modeling to contract valuation, risk management and other Mark to Market (MTM) activities. From the perspective of valuating generation assets, the production cost modeling approaches fails to provide support for the application of real option analysis.

A new modeling approaches for valuating generation assets within re-regulated electricity markets is needed. Such a model should incorporate both engineering and market insights. The model should be designed to meet the requirements of capturing the operation flexibility of generation assets using real option analysis. This work proposes a hybrid model to be discussed in Chapter three.

2.3 Short-Term Generation Assets Valuation

2.3.1. Application of Financial Option Theory

Re-regulation of electric power industry removes the obligation for GENCOs to serve, but awards GENCOs the flexibility to operate their generation assets. Traditional Discount Cash Flow (DCF) methods, such as Net Present Value (NPV), ignore the inherent flexibilities in operating generation assets, which renders them fundamentally flawed and under-valuate generation assets [22][23].

This section reviews the application of financial option analysis in valuating generation assets. More details on the theory of financial options could be found in [7]. Financial option theory models generation assets as call options on spark spread [12][13]. A spark spread is defined as the electricity price less the product of the generator heat rate and fuel price. Since this option involves two commodities with different market prices, its value depends on both prices and can be written as follows:

$$Value\_of\_Power\_Plant = max(P_E - HeatRate * P_F, 0)$$
$$HeatRate = ElectricityGenerated / UnitFuel$$

Equation 4: Call Option on Spark Spread

The financial option modeling captures the operating flexibilities of generation assets. However, modeling generation assets as pure financial assets leads to over valuation. The reason for over-valuation is that the spark spread option model ignores many features of real assets, such as physical constraints, market structure, and price movement. The direct impact of ignoring those features is that the option's value is forced to be non-negative by modeling, and that means the owner of such an asset will never loses money. Real option theory has been proposed to valuate generation assets, and the next section reviews researches on the application of real option analysis.

2.3.2 Application of Real Option Analysis

Real option analysis has been under intensive research and has seen successful applications in different industries such as oil, pharmacy, and hi-tech industries [22][23]. Real option analysis has been shown to give better results than DCF approach, especially when significant uncertainties and operation flexibility present. Real option analysis has seen more and more applications on the valuation of generation assets because it explicitly accounts for the flexibility in operating generation assets [14][15][16][17]. Real option provides GENCOs with an effective methodology to fully valuate the operating flexibilities, which can be flexibility inherent in the nature of generation assets or flexibilities traded on markets.

Real option analysis' superior performance for modeling and valuating uncertainties comes from its inheritance of financial option theory. Real option analysis overcomes the over-value drawback of modeling generation assets using financial options by incorporating operation constraints of generation assets.

The physical constraints of generation assets include time and capacity constraints. The capacity constraints state the volume of call option available to the owner of a generation

asset. Startup and shut down time state that a generator cannot be started up or shut down immediately, which means the decision to exercise a spark spread call option must be made before the prices on electricity and fuel are observed. Minimum on and down time states a generator must remain on or off once being started up or shut down, which means spark spread options are not always available and losses are possible. Ramp on rate limits the increase of a generator's output during start up, while ramp down rate limits the capability for a generator to decrease its output, both mean that a spark option is not always available for the full generation capacity.

There are arguments on the impact of physical constraints on values of generation assets, especially the impacts of ramp up and ramp down constraints. Deng states that start up cost, ramp-up time and output dependent heat rate has less impact on relatively efficient power plants like gas-fired power plants, thus could be ignored [14]. Tseng investigated the ramp constraint of power plants, and concluded that ramp constraints have impact on thermal power plants by reducing fuel economy, heat-electricity transformation efficiency and available generation capacity, which could be sold into spinning reserve markets as ancillary services [24].

Real option analysis should also recognize the behavioral differences between financial instruments and physical commodities. This work proposes to incorporate not only the physical constraints on operating generation assets but also their impacts on the electricity market movements. This research proposes an integrated modeling and valuating framework to fully employ the strength of real option analysis.

2.4 Investment on Generation Assets

Prior to re-regulation, generation planning or generation expansion aimed to lower production cost while maintain generation reserve margin no lower than the reliability threshold. Re-regulation replaces the centralized regulated utilities generation planning with re-regulated electricity market mechanisms monitored by ISO/RTO.

Krasenbrink et al. pointed out that planning tools that are able to support an integrated planning of power generation and trading is needed **Error! Reference source not found.**.

Integrated resource planning has been extended from regulation environment to re-regulation, and it aims at application from a social welfare perspective assumed perfect information [26][27]. Multiple criteria decision-making is also extended to competitive electricity markets with modified objective or constraints such as environment risks and utilizes multiple scenarios for optimization [28]. While some planning models assume price-taker behavior, game theory has been applied to capture the interaction between GENCOs, and one-round Cournot model was deployed [29]. Genetic Algorithm has been applied based on game theory approach, and improved the Cournot model by allowing agents to learn from each other and introduced some structure to the behavior of a GENCO [30]. The correlation of fuel market and electricity market has been explored by integrated planning for the natural gas and electricity systems [31].

Past researches focuses more on generation planning from a system-wide perspective and discusses the aggregated generation resources adequacy. Deng compared the valuation of generation assets based on real option analysis with the market transaction amounts on generation assets [14]. The simple assumption that multiplying short-term values could valuate long-term generation investment does not hold in electricity markets. The most-recent over-build and mothballing in generation capacity in several North America electricity markets suggests that a better tool for valuating investments in generation assets is needed. This planning tool should incorporate the dynamics of generation reserve margin and the dynamics of a specific generation asset' competition position within generation stack.

2.5 Missing Parts from Past Research

Applying real option analysis to valuate generation assets explicitly take the operation flexibility into consideration and has been shown to provide valuable insights. However, several features are missing from past research. The missing features have different impacts on the values of generation assets. In general, ignoring any forces limiting the operation flexibilities of generation facilities or surpassing uncertainties results in over-value while ignoring any forces enhancing the operation flexibilities or boost uncertainties results in

under-value. The reason is that value is created when uncertainties presents with flexibilities, which composes an option.

Past researches failed to incorporate information efficiently. The MTM approach assumes a static statistical structure and utilizes only historical market data, which were generated by fast evolving electricity markets. The engineering based production cost modeling approach ignores the significant market volatilities observed and assumes market equilibrium.

One feature missing from past research is the ignoring of market structure of an electricity market. Re-regulated electricity markets do not share the same feature with financial instruments markets. The differences in markets structure include market players' composition, trading liquidity, production, transportation and storage technologies among others. The most important feature of electricity market structure is the ownership of generation capacities. The dynamic unit commitment of electric power system in fact leads to time-varying generation capacity ownership compositions. The dynamics of generation capacity ownership segments electricity markets into peak and non-peak time intervals. The instantaneous balance between electric energy supply and demand, limited transfer capabilities of transmission networks and non-existence of large-scale economic storage of electric energy segment electricity markets into different zones. The physically homogeneous electric energy is financially heterogeneous, demonstrating time-varying and location-dependent values.

Electricity markets' architectures are also ignored in past researches. Although the electric energy is the staple of electricity markets, there are many other markets such as ancillary services, capacity, emission and others. Most of the services could be provided from the every same generation assets as of electric energy. A GENCO might have the operating flexibility to choose which sub-markets to supply, optimizes the operation of its generation assets.

Past researches simplify the options available to a GENCO by assuming a GENCO to be a price-taker on a single forward electric energy market and a GENCO makes its decision solely based on the information on electric energy markets. The operation of generation assets becomes passively responding to the only signal sent from a single electric energy

market. However, each GENCO actively optimizes its portfolio including both financial and physical assets. A valuation approach ignoring the active portfolio management of generation assets leads to under-valuation.

Past research fails to model the market evolution, which link short-term market models into long-term markets models. The observed generation assets building heat and cold off are partly due to the lack of a long-term risk share mechanism and decision-making support tools for GENCOs. An appropriate investment tool should allow the modelers to incorporate subject judgments for the long-term market forecast is highly uncertain so that objective forecast is of lower quantity.

Chapter 3: Modeling of Electricity Markets

This chapter begins with analyzing the electricity markets and the features observed on the electricity price movements. The deficiencies of past research and needs on modeling electricity markets are also identified in Section 3.1. Section 3.2 introduces the concept of Hidden Market Model (HMM). Section 3.3 proposes to model a general electricity market with HMM. Section 3.4 introduces the concept of Location Marginal Price (LMP). Section 3.5 extends HMM to a special application, the modeling of LMP. Section 3.6 proposes to extend the short-term model discussed into mid-term and long-term modeling of electricity markets. Fuzzy sets and numbers are employed to capture the extreme uncertainties associated with mid/long-term electricity markets movements.

3.1 Electricity Markets as Special Commodity Markets

As commodity markets, electricity markets' structures and architectures are significantly different with financial markets. The market structure and architecture are of critical importance of understanding and modeling electricity markets' movements. Chapter one discusses electricity markets' structure and architecture. While chapter one focuses more on the composition of electricity markets, this section focuses on the dynamics of electricity markets.

The physically homogeneous electric energy is heterogeneous temporally and spatially. One significant feature of electricity markets is the complicated seasonality, showing pattern in different time horizons. Three factors contribute to the yearly, weekly, and daily patterns observed on electricity markets:

- The seasonality of demand for electric energy
- The indispensable nature of electric energy as a commodity
- The technical constraints on supplying and storing electric energy

Based on the nature of demand and supply of electric energy, a regional electricity market can be segmented into base-load, intermediate-load and peak-load sub-markets shown in Figure 8.

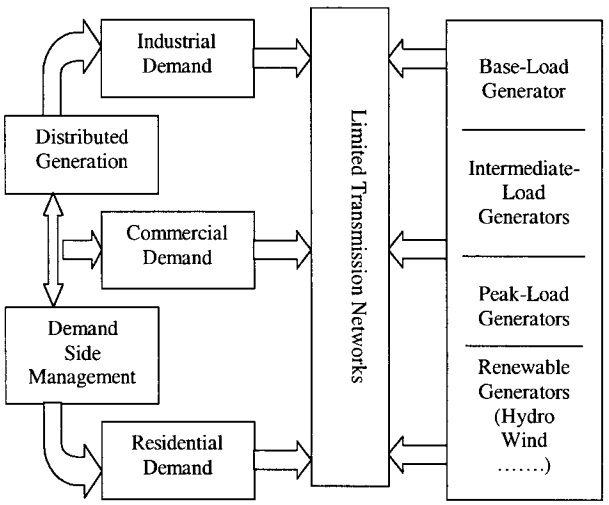


Figure 8: Temporally Segments of Electricity Markets

Base-load markets, intermediate-load markets and peak-load markets compose cyclic electricity markets movements as shown in Figure 9.

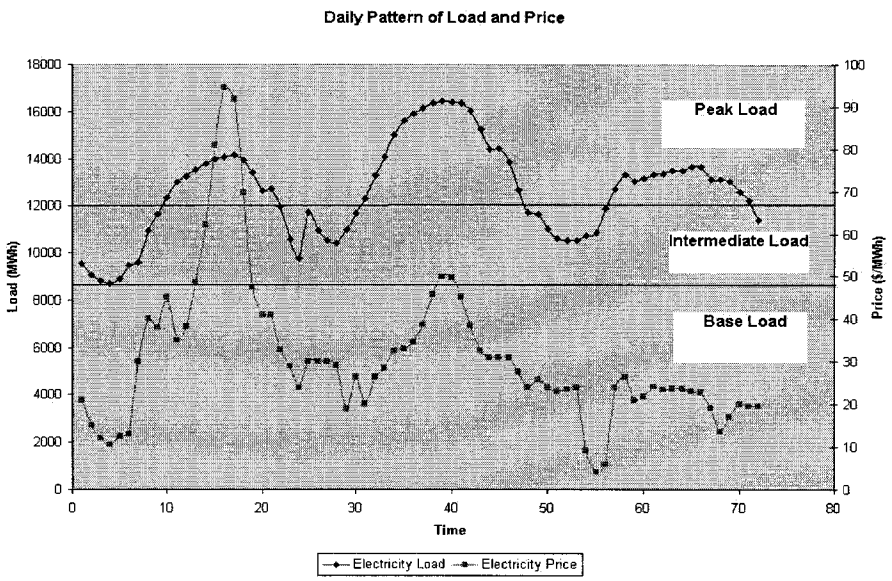


Figure 9: Dynamics of Base, Intermediate and Peak Load

Each sub-market has different market players. For base-load electricity markets, the key suppliers are base-load generators that utilize similar technology to achieve efficiency of producing electricity. Most often, the base generators are nuclear generators, combined cycle combustion turbine units burning natural gas, and ultra–temperature and pressure coal burning generators with generation capacity ranging from 500MW to 1000MW and more. Base generators in general are slow units in the sense of starting up/shutting down/ramping up/down. The transmission network is less stressed during base-load periods, thus the base-load electricity market is geographically more spanned.

Peak-load generators share similar technology to achieve fast start up and shut down capability. Most often, the peak generators are natural gas generators with capacity ranging from 50MW to 200+ MW. Peak generation units in general are less efficient than base generators, but takes much less time to change their operating statuses. The transmission network is more stressed during peak-load periods, thus the network is more easily congested during peak-load periods.

Intermediate generators' efficiency and speed of changing operating statuses are between based generators and peak generators. The same is true of the transmission networks during intermediate load period.

Renewable generators could not easily be categorized based on the load they serve. The reason is that most renewable generators have little control on their raw energy sources. Such generators include wind generators, run-of-the-river hydro generation units, solar generators and earth-thermal generators. One exception is the hydro generators with reservoirs, which enables the hydro generation unit the operating flexibility to supply electric energy. With further development of renewable energy, the electricity markets movements expect significant changes.

Electricity demand could be categorized into industry, commercial and residential customers based on the objectives and technologies of electricity consumption. Base demand includes invariant parts of all three-load groups, which depends mainly on population size and macroeconomic trend. Peak electric demand is due to commercial and residential

customers for heating/cooling and therefore highly correlated with weather. Intermediate electric load are often referred to as the transition between base and peak load periods.

The difference in the supply and demand forces in base-load, intermediate load and peak-load electricity markets suggests different market participants, structures and behaviors. Physical system drivers also play significant roles in driving electricity markets. During peak electric load period, the infinite-transfer capability of electric power transmission network segments an integrated electricity market into several zones with heterogeneous markets structure. The heterogeneous economic and physic drivers lead to different market prices movements.

Past researches ignores the heterogeneousity and dynamics of electricity markets, assuming a single stationary electricity market driven by homogeneous drivers. This work aims to capture the dynamics of market structure, both temporally and spatially.

This work proposes to model electricity markets as dynamics systems. The system is defined by a closed system states space. The transition is approximated with Markov processes, driven by physical and economic drivers. Markov processes capture the dynamics of both the physical and economic drivers.

Physical constraints of generation units link the generation production level of one hour to a previous hour. The aggregated online generation capacity at the next time interval could be modeled as dependent on the current level of online generation capacity and the unit commitment and dispatch decisions of GENCOs. Elliott models the operation state of generators with Markov chain model and analyses the prices jump at Alberta electricity market [32]. Dynamics of electric power load could also be approximated by Markov process [33].

Markov processes are also consistent with weak-form market efficiency, which is often assumed by Mark-to-Market approach. Price on Electricity has been modeled to follow Geometric Brownian motion, which is a Markov process. Some researchers have proposed to divide the movement of price on electricity into different regimes, where each regime has its own stochastic model for price movement. [33][35] The switching between different regimes is modeled as a Markov chain. C. C. Liu modeled a GENCO's bidding strategy

using Markov chain model, where a GENCO bids low, middle or high with its state transit following a Markov chain model [36].

3.2 Hidden Markov Model

3.2.1 General Markov Model

A stochastic process is defined to be an indexed collection of random variables {X}, where index runs though a given set $\{T\}$. If a stochastic process has the Markovian property, it is qualified to be a Markov chain or Markov process [37]. A Markov chain or Markov process consists of a given set of states and the transition matrix as shown Equation 5.

Markov Chain Defintion

Transition Probability Matrix P, (stationary, time - varing, ...)

States, (Limited, Unlimited...)

$$\begin{array}{ccccccc}
Time, t=0 & & Time, t=1 & & Time, t=T-1 & & Time, t=N \\
State_1 & & State_1 & & State_1 & & State_1 \\
State_2 & \xRightarrow{P^{t=0}} & State_2 & \xRightarrow{P^{t=1}} & State_2 & \xRightarrow{P^{t=T-1}} & State_2 \\
. & & . & & . & & . \\
State_N-1 & & State_N-1 & & State_N-1 & & State_N-1 \\
State_N & & State_N & & State_N & & State_N
\end{array}$$

Equation 5: Definition of Markov Processes

The Markovian property says that the conditional probability of any future event, given any past event, and the present state, is independent of the past event and depends only on the present state, as shown in Equation 6.

Markovian Property

$$P\{X_{t=1} = j \mid X_0 = k_0, X_1 = k_1, \ldots X_t = i\} = P\{X_{t=1} = j \mid X_t = i\}$$

for any t=0,1 .. and every sequence i, j, $k_0, k_1 \ldots$.

Equation 6: Markovian Properties

Markov chain models have also been applied in financial area, namely approximation of the distribution and movement of underlying asset value in numerical methods on pricing derivatives. Compared with multinomial trees, Markov chain model is shown to more flexible and efficient [38]. For options pricing with binomial trees, the length of a time step and the number of asset prices generated by the tree are simultaneously determined for a particular maturity. For the Markov chain method, the length of a time step and the number of discrete asset prices are independently set. Markov chain's closed set of system states reduces the discrete prices need to be generated, thus increases efficiency. While binomial tree shows a typical jagged convergence pattern, Markov chain method converges more smoothly and faster.

3.2.2 Hidden Markov Model

3.2.2.1 Basics of Hidden Markov Model

If the states of a Markov chain are not fully observable, we have Hidden Markov Model (HMM). A pair of stochastic processes (X; Y) is a Hidden Markov Model (HMM) if X (the state process) is a Markov process and Y (the observable process) is an incomplete observation of X [39]. The observation can be deterministic or probabilistic and the observable can be a state or a state transition. Mathematically, HMM is a doubly embedded stochastic process with an underlying stochastic process that is not observable, but can only be observed through another set of stochastic processes that produce the sequence of observations. Equation 7 defines a HMM with discrete observable variables.

Two Model Parameters :

N (Number of States)

M (Number of distinct observation symbols per state)

State Transition Probability Matrix A

Observation Symbol Distribution given a State B

Initial State Distribution, p

$A : a_{ij} = P[q_{t+1} = S_j \mid q_t = S_i], 1 \le i, j \le N$

$B : \mathrm{b_j}(k) = P[v_k \text{ at t} | q_t = S_j], 1 \le j \le N, 1 \le k \le M$

$\pi : \pi_\mathrm{i} = p[q_1 = S_i], 1 \le i \le N$

Equation 7: Hidden Markov Model

Several features of HMM make it a good candidate for theoretical analysis. First, HMM has a very rich mathematical structure such that a sequence of HMMs of increasing size can approximate any ergodic stochastic process in the weak and cross entropy sense [40]. The second merit of HMM is its strength to explain extreme variations in the observed process based on a postulated hidden process. In particular, a HMM attributes this over-dispersion to the key model feature that observations come from one of several different marginal distributions, each associated with a different latent state. An HMM is capable to capture the over-dispersion in the observed electricity market data by showing that the markets are transiting among different states. The third merit is that there are very efficient algorithms to solve (Forward-Backward Method) HMM. More details on HMM can be found in references [41][42][43][44]. Finally, HMM can be extended to many special cases, which have potential application in modeling electricity markets.

3.2.2.2 Extensions of Hidden Markov Model

The understanding of HMM has advanced considerably since the realization that HMM is a kind of Bayesian Networks (BN), more specifically a special case of Dynamic Bayesian Network (DBN) [43]. HMM can be extended to more complex and interesting models such as FHMM, Tree-Structured HMM, and Switching State Space model. General solutions to the problems of parameters learning, and model selection have been under

intensive research and efficient algorithms are available for some special cases. The following section discusses Bayesian Networks, Dynamic Bayesian Networks, and general solutions to both BN, DBN and their extension to HMMs.

A Bayesian network is a graphical model for representing conditional independencies between a set of random variables, as shown in Figure 10. A Bayesian Network, also known as belief network, probabilistic graphical mode or probabilistic independence network is a marriage between probability theory and graph theory. It provides a natural tool for dealing with two problems that occur throughout applied mathematics and engineering, uncertainty and complexity.

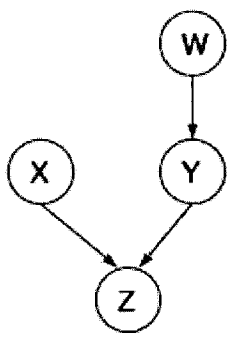


Figure 10: Bayesian Network

Fundamental to the idea of a graphical model is the notion of modularity; a complex system is built by combining simpler parts. Probability theory provides the glue to integrate different parts, ensuring that the system as a whole is consistent, and providing ways to interface models to data. The graph models provide both an intuitively interface to modelers and a data structure that lends itself naturally to the design of efficient general-purpose algorithms. Many of the classical multivariate probabilistic systems studied in fields such as statistics, systems engineering, information theory, pattern recognition and statistical mechanics are special cases of the general graphical model formalism, such as mixture models, factor analysis, Hidden Markov models, and Kalman filters. The graphical model framework provides a way to view all of these systems as instances of a common underlying formalism. This view enables to apply specialized techniques, which have been developed in one field, to be transferred between research communities and exploited more widely.

HMM falls in a subclass of Bayesian Networks known as Dynamic Bayesian Networks, which are simply Bayesian Networks for modeling time series data. In time series modeling, the assumption that an event can cause another event in the future, but not vice-versa, simplifies the design of the Bayesian network: directed arcs should flow forward in time. Another very well known model in this class is the Linear-Gaussian State-Space model, also known as the Kalman filter, which can be thought of as the continuous-state version of HMM. What makes HMM and State-Space Models (SSM) special is that their hidden state spaces are closed under their respective state transition probabilities and output models. This closed property of HMM and SSM makes inference and learning particularly simple and appealing in these models, such as Maximum Likelihood (ML) in HMM and Kalman filtering in SSM. However, those methods can only crudely approximate Bayesian learning, and will perform catastrophically when data is scarce and/or the model is complex.

- Factorial Hidden Markov Models (FHMM)

An HMM is essentially a mixture model, encoding information about the history of a time series in the value of a single multinomial variable, the hidden system state. This multinomial assumption allows an efficient parameter estimation algorithm to be derived, the Baum-Welch or EM algorithm. However, it also severely limits the representation capacity of an HMM. For example, to represent 30 bits of information about the history of a time series, an HMM would need 2^{30} distinct states. On the other hand, an HMM with a distributed state representation could achieve the same task with 30 binary variables.

The distributed state representation incorporates flexibility into HMM, and increases the modeling power. First, such representation decomposes the state space into features that naturally decouple the dynamics of the process generating the time series. In this study, demand and supply on electricity determine the market clearing price and quantity together. Although the relationship between demand and supply can be modeled as system states such as under-supply, equilibrium, over-supply; a more natural approach is to model the demand and supply sides as distributed system states. Also, the distributed representation facilitates the incorporation of ancillary services into an electricity market model.

Secondly, distributed state representations simplify the task of modeling time series generated by the interaction of multiple independent process. In this study, it allows modeling market architectures into the market model by explicitly defining how the observations depend on incompletely observable states. Ghahramani defines HMMs with distributed state as Factorial Hidden Markov Models (FHMM), and Figure 11 compares HMM with FHMM [42].

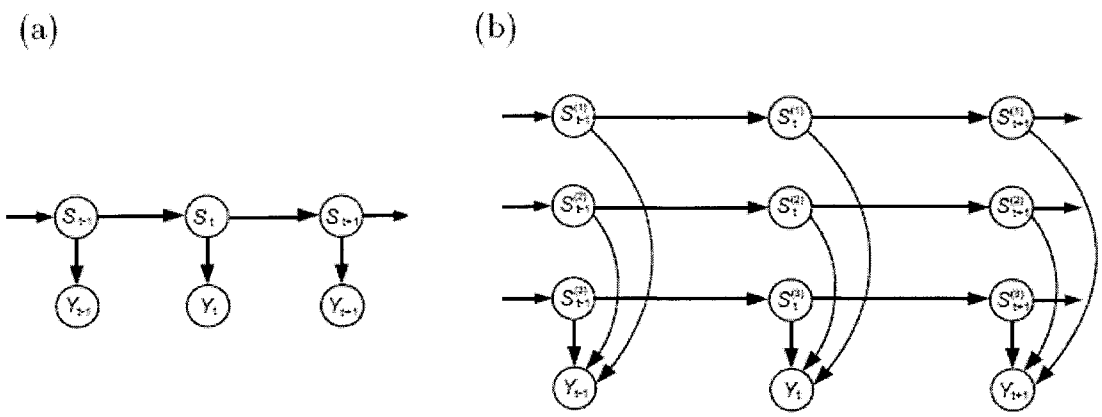


Figure 11: Comparison between HMM and FHMM, (a) HMM (b) FHMM

S is defined as hidden system state while Y is defined as observation or emission

- Dynamic Bayesian Network (DBN)

When HMM is extended to FHMM, exact algorithms for parameter learning and model selection become intractable and new approximate algorithms are based on general solutions to Dynamic Bayesian Networks. Modeling FHMM as DBN provides advantages because that a DBN may have exponentially fewer parameters than corresponding FHMM thus inference in a DBN may be exponentially faster than in the corresponding FHMM. For a FHMM with D chains, each with K values, the numbers of parameters to define the probability $P(X_t \mid X_{t-1})$ are of $O(K^{2D})$ for FHMM and $O(DK^2)$ for DBN. The computational complexity of exact inference is of $O(TK^{2D})$ for FHMM and $O(TDK^{D+1})$ for DBN. Since DBN is more general than FHMM, its inference algorithms can also be applied

to other variants of HMM. In Figure 12, Input-Output HMM might be of interest if the demand for electricity is modeled as input to an electricity markets. This is possible and interesting because that ISO normally forecast and announce electricity demand to the public, and currently demand for electricity has no short-term self-price elasticity [45].

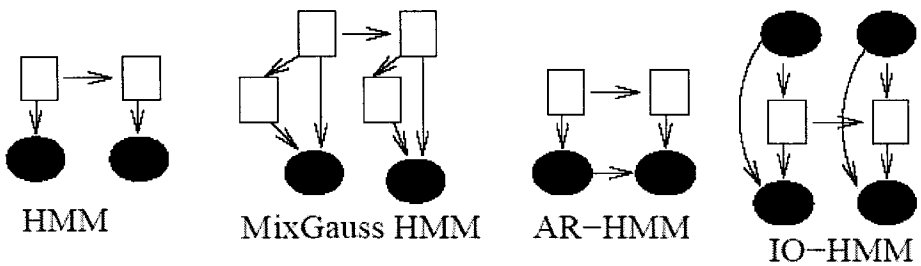


Figure 12: Variants Implantations of DBN

Shaded shapes are observable variables while empty shapes are hidden states

- Tree-Structured HMM, Switching State-Space Models and Other Extensions

Other extensions of HMM include Tree-Structured HMM, Switching State-Space Models, and they have potential applications in modeling electricity markets. In FHMM, the state variables at the same time slice are assumed to be independent given the state variables at the previous time slice. This assumption can be relaxed in many ways by introducing coupling between the state variables in a single time slice. In modeling electricity markets, the demand for ancillary services could be modeled as depending on demand on electric energy. Figure 13 illustrates the structure of a Tree-Structured HMM, where X defines input, S defines hidden states, and Y defines output. The architecture can be interpreted as a probabilistic decision tree with Markovian dynamics linking the decision variables, which could be the temporal constraints for demand and supply on electricity.

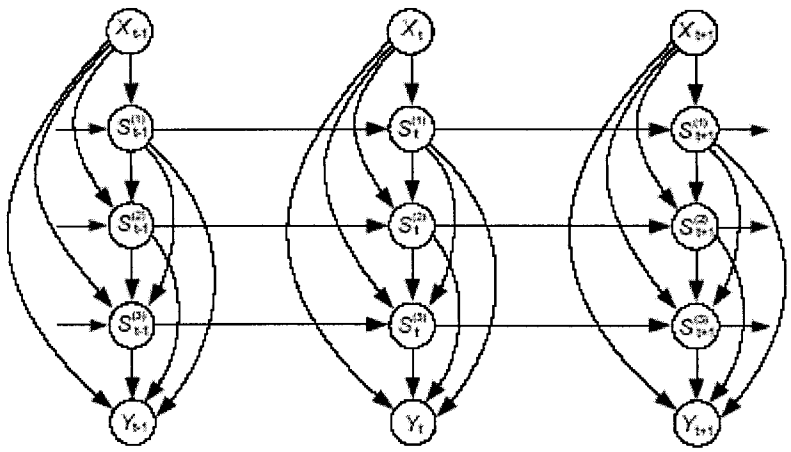


Figure 13: Tree-Structured HMM

At any time interval, the input *X*, which is the forecasted demand for electricity by ISO, enters the electricity market model. The demand for electric energy $S^{(1)}$ also determines the demand for ancillary services $S^{(2)}$, while the supplied generation capacity $S^{(3)}$ can be used to meet demand for electric energy, ancillary service, or both.

The modeling of an electricity market should allow the states to be continuous variables, which is more natural for demand on electricity. This can be achieved by extending HMM to Switching State-Space Model (SSSM). Figure 14 illustrates the structure of SSSM. In SSSM, the observations *Y* is modeled using a hidden state space comprising *M* real-valued state vectors *Xs* and state discrete state vector *S*. In modeling electricity markets, *X* could be real-valued demand and supply capacities on electricity, while *S* defines how the market prices *Y* depends on *X*. For example, *S* could be a binary state variable defines whether the transmission networks are congested or not.

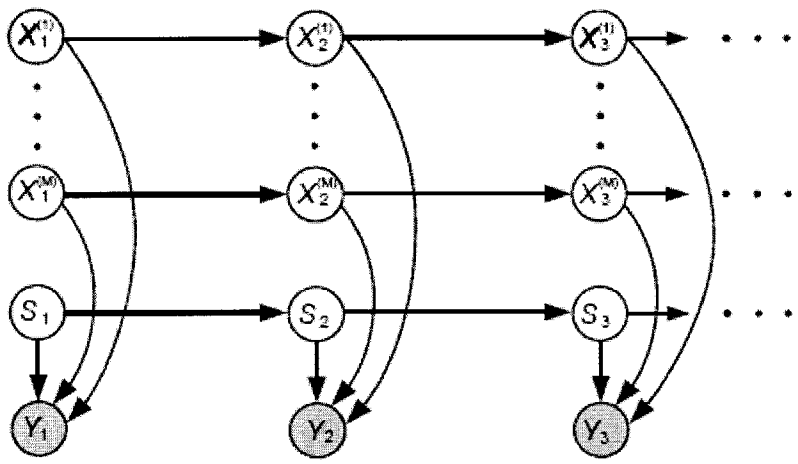


Figure 14: Switching State Space Model

3.2.2.3 HMM Estimation and Numerical Problems

Algorithms to estimate FHMM and other extensions are only approximations to the exact algorithm for tractability. Two main approaches are sampling methods and variation methods. The dominant sampling method is Monte Carlo Markov Chain (MCMC), which allows us to draw samples from a Markov chain instead of from the posterior distribution $P(X \mid Y)$. The Markov chain will be updated and is guaranteed to converge to the posterior probabilities of the states given observations as long as none of the probabilities in the model is exactly zero. The simplest case of MCMC is Gibbs sampling method, which is implemented by Ghahramani [42]. The variation methods essentially decouple all the nodes in DBN, and introduce a new parameter, called a variation parameter for each node, and iteratively update those parameters so as to minimize the cross-entropy (KL distance) between the approximate and true probability distributions. Updating the variation parameters becomes a proxy for inference and produces a lower bound on the likelihood.

Other numerical problems associated with FHMM and DBN include overfitting and model selection. The following section discusses those two subjects briefly and more details could be found [39]. Overfitting refers to the scenario where the estimated model fits the training set very well but generalizes poorly to a test set chosen from the same data distribution. It is most prevalent when the training set is small relative to the complexity of

the model and there is nothing in the maximum likelihood fitting procedure itself can avoid it. Three approaches are available now, cross-validation, regularization, and Bayesian integral. Model selection, or learning model structure, is closely related problem of picking a particular structure amongst several alternatives. For example, a modeler need to choose either a regular HMM or extension to HMM or parameters such as the number of hidden states in a model before parameter estimation. Sometimes, the detection of missing hidden system states can be critical to the performance of a model. There is nothing in ML parameters fitting that does this automatically. There are some algorithms for model selection, but requires human being's intervention. Both overfitting and model selection require the judgments of a modeler, where the fitness of a model not only depends on its statistical or computational performance but more importantly, its explanation power for the observation based reasonable logics. Those subjects will be discussed when it presents itself in this study.

3.2.2.4 Markov Decision Process and Partially Observable Markov Decision Process

When the decision-maker could have impacts on the state transition, Hidden Markov Model (HMM) evolves into Partially Observable Markov Decision Process (POMDP). For a HMM, a decision-maker is a "state taker", who observes and forecasts the state transition and make decision based on its perception. A winder generator is most often a state taker confronting a HMM. For a POMDP, a decision-maker actually has some control over the transition of states, thus a "state maker". If an electricity market is only partially observable, a hydro electricity station with reservoir, where the remaining water level is under control of a GENCO, could be modeled using POMDP. Table 2 illustrates the concepts of Markov Process, MDP, HMM and POMDP.

Table 2: Markov Process, MDP, HMM and POMDP

Markov Model		States Transition Controllable	
		NO	YES
States	YES	Markov Chain	MDP

Observable	NO	HMM	POMDP

The following discusses the difference between Markov Decision Process (MDP) with POMDP, and the solution to POMDP. In an MDP, the state of the world is completely observable. A policy is a mapping from the set of states, which are oberservable, to the set of actions. If both sets are assumed to be finite, the number of possible mappings is also finite. An optimal policy can be found by conducting search over this finite set of mappings. In a POMDP, on the other hand, the observations do not provide sufficient information about system states. Information from previous steps need to be taken into consideration. All such information can be summarized by a probability distribution over the set of states if Markovian property exists. In the literature, this probability distribution is often referred to as a *belief state*. The *belief space* is defined to be the set of all possible belief states. It is a continuous space although the world has only a finite number of states. In a POMDP, a policy is a mapping from the belief space to the set of actions. This definition seems similar to a policy in the MDP context. However, the continuum of the belief space poses a challenge from the computational perspective because the number of mappings can be uncountably many. To find an optimal policy for a POMDP, one has to conduct search over this space. This fundamental difference makes solving POMDPs drastically more difficult than solving MDPs.

Closely related to the concept of a policy is the concept of a value function, which is a mapping from belief states to real numbers. Each feasible policy has an associated value function and the better the policy, the better its value function. On the other hand, one can construct a policy given a value function and the better the value function, the better the policy constructed. This leads to another strategy to solve a POMDP, i.e., to conduct search in a value function space. Value iteration does exactly this: it improves value functions in an iterative fashion. Each iteration is referred to as dynamic-programming (DP) update. A DP update computes a new value function over the entire belief space from the current one. Value iteration stops when the current value function is sufficiently close to the optimal. Since each iteration needs to consider uncountably many belief states, DP updates are expensive. Moreover, value iteration needs to conduct many steps of DP updates before it

can find a near optimal value function. These two factors make value iteration very inefficient. This has led to the development of new exact algorithms to efficiently compute optimal solutions, and various approximation approaches.

A POMDP can be solved exactly or approximately by value iteration algorithm [45], enumeration algorithm [46], one-pass algorithm [47], linear support algorithm [48], witness algorithm [49], incremental pruning algorithm [51], and other algorithms. All the algorithms are based on dynamic programming. Special features of electricity markets and geneation assets valuation could be utilized to improve the performance of solving a POMDP.

3.3 Modeling Electricity Markets with HMM

This section proposes to model the movements of electricity markets as HMM driven by underlying economic forces. An electricity market is modeled as a dynamic system evolving over time according to Markov processes. At any time interval, the electricity market can be in one state and transit to another state in the next time interval. The states of an electricity market are modeled as partially observable, while each state has incomplete observations such as market-clearing price and quantity. The true market states are hidden from a market participant behind the incomplete observation. HMM is of a more fundamental approach and focuses on capturing the interaction of supply and demand forces on electricity markets. An example is given to apply HMM to historical data from New York Independent System Operator (NYISO).

The simpleast form of HMM is regime switching. Regime switching states that there are unobserved market regimes following Markov processes underlying the observed price movements. Often, electricity markets are categorized into stable regime with less volatility and unstable regimes with extreme volatility. For each regime, an individual econometric model is proposed and fitted with historical data. Figure 15 Illustrates a regime switching diagram.

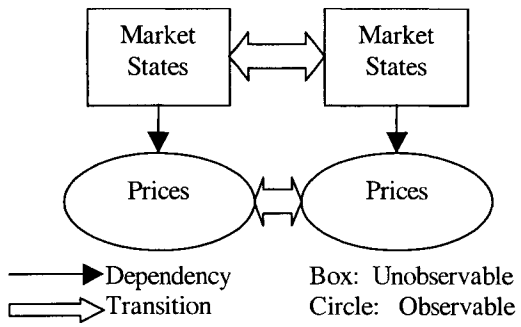


Figure 15: Regime-Switching, the Simplest HMM

The simplest HMM shown in Figure 15 encodes information of a time series with the value of a single multinomial variable, the hidden system state. HMM could be extended to FHMM, which is capable to decompose market drivers such as demand and supply forces. This feature allows incorporating market architecture into the market model by explicitly defining how the observations depend on incomplete observable states. An HMM with distributed states is defined as a Factorial Hidden Markov Model (FHMM), shown in Figure 16. The electricity markets states are defined by three factors, demand on electric energy, demand on ancillary services and online generation capacity. Demand on ancillary service is further modeled to be dependent on demand on electric energy. This model decomposes markets into different factors/drivers, and imposes more structural features.

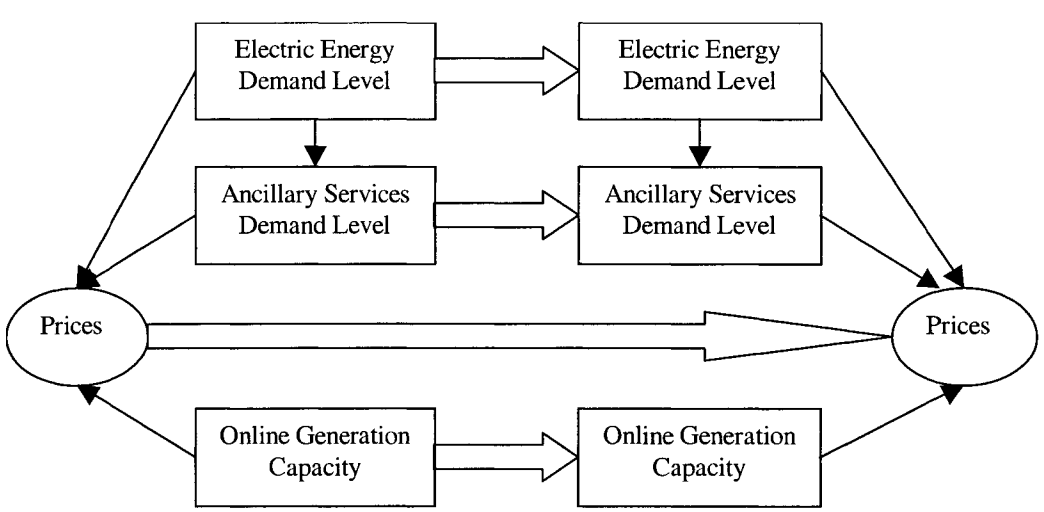


Figure 16: FHMM for Electricity Markets

In FHMM, the state variables at a time interval are assumed to be dependent only on the corresponding state variables at the previous time interval. This assumption can be relaxed by introducing coupling between the state variables in different time intervals. Tree-Structured HMM (TSHMM) allows modeling of the interdependence between state variables of different time intervals. The TSHMM, shown in Figure 17, models the supplied generation capacities at time *T+1* to be dependent on the demand on electricity at time *T*. This feature captures mean-reversion, which is the results of behaviors of generation companies to turn on more generators when higher prices are expected.

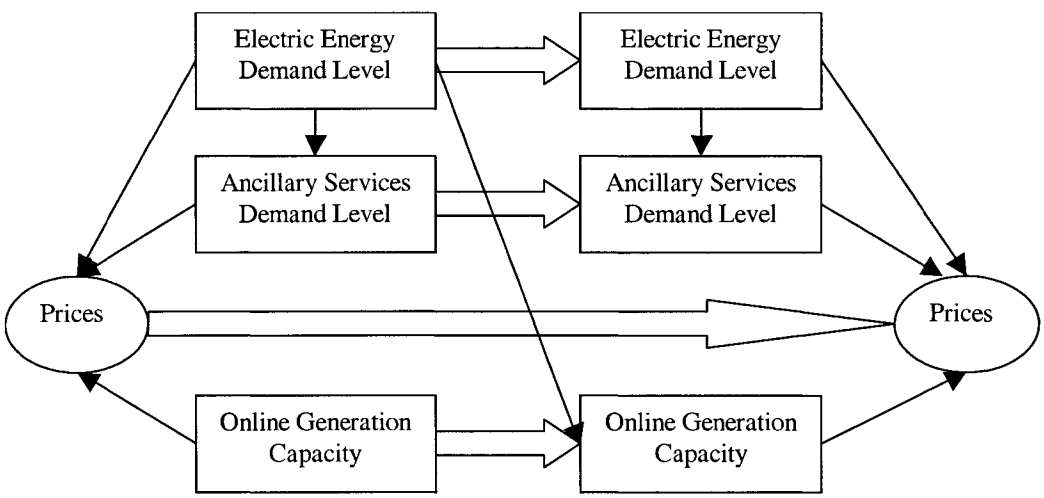


Figure 17: Tree-Structure HMM for Electricity Markets

The Switching HMM (SHMM) allows multiple dependencies to be dynamically chosen by a switch variable. The SHMM as shown in Figure 18 incorporates the impact of transmission networks. The congestion of transmission networks has significant impacts on how the electricity prices will be determined by the supply and demand forces.

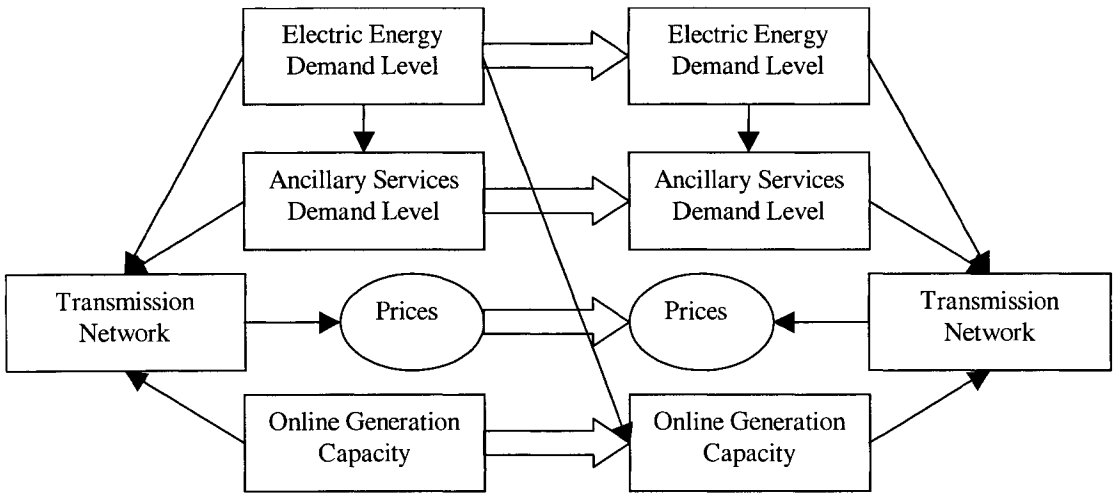


Figure 18: Switching HMM for Electricity Markets LMP Decomposition

Other extensions of HMM are also possible and see potential applications in modeling electricity markets. In addition, electric markets can be modeled with multiple HMMs. Each HMM approximates a time-segment of the electricity market such as base-load, intermediate-load and peak-load markets. Multiple HMMs are linked together by the system states. The ending states of the last HMM are the initial states of the next HMM in the chain.

HMM combines electricity market structure, architecture, and competition strategies of market players into one integrated mathematical framework. The electricity market structure and architecture determine the structure of an HMM such as how many market states exist and the definitions of a market state. The economic position of a market player determines its competition strategies, which in turn jointly determines how an electricity market evolves from one state to another, namely the transition matrix. The randomness caused by unintended forecast errors is defined with the distribution of prices for each specific market state. The following example estimates an HMM with historical data from NYISO.

This example models an hourly electric energy market with the simplest HMM, regime switching, having three market states where each has a discrete distribution for observable prices. The markets states are defined based on the relationship between supply and demand for electricity. For a given level of demand on electricity, there are three

possible levels of online generation capacity and thus different market states defined in Equation 8.

If the second most expensive generator is operating at its partial capacity, the market is assumed to be in "Punch-In" state, which means that generators are trying to be get dispatched and might be suffering loss. The incentive for a GENCO to operate at such a level is that a GENCO is expecting higher margins in the coming market hours. The reason forces a GENCO to operate at such a level is that there are time-constraints on a generators' operating. In order to be able to operate at its full capacity at higher prices of peaking hours, a generator must get dispatched and ramp up its capacity according to its time constraints. Other possibility is that a GENCO is shutting down after peak hours, but could not reduce its generating level fast enough. "Punch-In" market states often occur during the hours immediately before peaking hours, for example, 8AM – 10AM or hours immediately after peaking hours, for example, 8PM-10PM. If the second most expensive generator is operating at its full capacity, the market is assumed to be in "Harvesting" state, which means that all generators except the marginal generator are already at their fully capacity at a price level above their production cost. However, the market is in equilibrium for the marginal generator is operating at its production cost and there is no incentives for other offline generator to punch-in the markets. "Harvesting" market states often occur during non-peak hours with relative slow and expected demand changes, for example, 10PM-8AM. If the marginal generator is operating at its fully capacity, the market is assumed to be in "Ripping-Off" state, which means all generators are operating at premiums. "Ripping-Off" market states often occur when the transmission network is congested or outages of generation. "Ripping-Off" market states are often observed with price spikes in pairs, for outages and congestions are often mitigated by the ISOs/RTOs.

$$MarketState = \begin{cases} Harvesting & if\ Cap(n) >= Demand\ >= Cap(n-1) \\ Punch-In & if\ Demand <= Cap(n-1) \\ Ripping-Off & if\ Demand >= Cap(n) \end{cases}$$

$$-Cap(n) = Capacity\ of\ all\ n\ online\ units$$

$$-Cap(n-1) = Capacity\ of\ (n-1)\ online\ units$$

$$except\ the\ most\ expensive\ unit\ (Marginal\ Unit)$$

Equation 8: Three-States Regime Switching

The distributions of observable prices given a specific market state are assumed to be discrete and have a daily price pattern as defined in Table 3. The true and estimated parameters for the market are also shown in Table 3, and simulated prices paths are shown in Figure 19. The Baum-Welch algorithm is employed to estimate the model, which converged in two iterations.

The market states transition matrix defines the probability for the market to transit from a given state to another. For example, the number "0.58" observed at the left-up corner defines the probability for the market to transit from state-1 (Punch-In) at time T to state-1 (Punch-In) at time T+1. The price distribution given market states matrix defines the probability of price to come from three levels (Low, Mid and High) given a market state at time T. For example, the number "0.18" observed at the left-up corner defines the probability for a "Low" price to be observed if a market is in state-1 (Punch-In). This example uses true parameters to simulate prices. The simulated prices are then used to estimate the true parameters assuming a guess defined as initial parameters.

Table 3: Three States Regime Switching Example

Market States Definition		State-1(S-1) Punch-In	State-2(S-2) Harvesting	State-3(S-3) Ripping-Off
Market States Transition Matrix				
		State-1	State-2	State-3
S-1	True Parameters	0.58	0.32	0.1
	Initial Parameters	0.2	0.8	0.0

	Estimated Values	0.5624	0.3254	0.1122
S-2	True Parameters	0.275	0.45	0.275
	Initial Parameters	0.05	0.9	0.05
	Estimated Values	0.2999	0.4228	0.2773
S-3	True Parameters	0.1	0.32	0.58
	Initial Parameters	0.0	0.8	0.2
	Estimated Values	0.1143	0.3071	0.5786
Price Distributions Given Market State				
		Lower Price	Normal Price	Higher Price
S-1	True Parameters	0.18	0.64	0.18
	Initial Parameters	0.7	0.2	0.1
	Estimated Values	0.1767	0.6386	0.1847
S-2	True Parameters	0.18	0.64	0.18
	Initial Parameters	0.1	0.8	0.1
	Estimated Values	0.1866	0.6323	0.1811
S-3	True Parameters	0.18	0.64	0.18
	Initial Parameters	0.1	0.2	0.7
	Estimated Values	0.1587	0.6484	0.1929

Daily Price Pattern

Time (Hour)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Price ($/MWh)	15	15	15	15	17	19	21	23
Time (Hour)	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Price ($/MWh)	25	27	30	34	40	50	40	34
Time (Hour)	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Price ($/MWh)	30	27	25	23	21	19	17	15

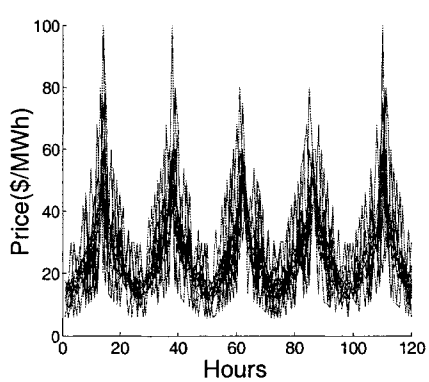


Figure 19: Simulated Price Paths from FHMM

The following example assumes and estimates a FHMM for the Day-Ahead houyly market in west control area of NYISO from April 7th to July 27th 2003 [52]. Day-Ahead market trades the bulk of electricity, therefore most reflective of the whole electricity markets. The FHMM includes prices on electric energy and ancillary services (30-minutes spinning reserve). The model estimation employs the Baum-Welch algorithm [44], and the estimated parameters are shown in Table 4.

Similar to the state transition probability shown in Table-3, Table-4 defines the probability of the electricity market transit from one state to another. The difference lies in that FHMM shown in Table-4 are driven by two factors: demand and supply. The up-left corner of "Demand on Electric Energy States Transition Matrix", "0.9229", defines the probability of a demand of level "Low" at time T to transit to level "Low" at time T+1. The supply transition probability is conditional on not only the previous supply state but also the demand level. The up-left corner of "Online Generation Capacity States Transition Matrix", "0.9585", defines the probability of supply of level "Low" to transit to level "Low" when the current demand is at "Low" level. The prices for any given market states are approximated by a normal distribution. The mean and variances of the normal distributions are shown in Table-4.

Only online generation capacity is considered as effective supply for physical operating constraints limit prohibit offline generation capacity to produce on short notice less than one hour. On current electricity markets, demand shows no self-price elasticity while the generators have control over its productions. The market observed clearing quantity is in fact the demand, but not the supply. The supply includes all the generation capacity "bid" into electricity markets, which are only partially observable. For the Day-Ahead markets, the physically online generation capacity at a specific hour does not necessarily equals to the capacity bid into markets, thereby hidden to a GENCO.

Table 4: FHMM Example

Demand on Electric Energy States Transition Matrix			
	Low	Normal	High
Demand at T = Low	0.9229	0.0771	0
Demand at T = Normal	0.0199	0.9634	0.0168

Demand at T = High	0	0.1085	0.8915
Online Generation Capacity States Transition Matrix			
If Online Generation Capacity at Time T-1 = Low			
Demand at T = Low	0.9585	0.0315	0.0100
Demand at T = Normal	0.9434	0.0466	0.0100
Demand at T = High	0.0000	1.0000	0.0000
If Online Generation Capacity at Time T-1 = Normal			
Demand at T = Low	0.2097	0.7803	0.0100
Demand at T = Normal	0.1531	0.8369	0.0100
Demand at T = High	0.5898	0.4002	0.0100
If Online Generation Capacity at Time T-1 = High			
Demand at T = Low	0.0000	0.9900	0.0100
Demand at T = Normal	0.0000	0.9900	0.0100
Demand at T = High	0.0000	0.9900	0.0100
Interdependency of Demand on Ancillary Service on Demand of Electric Energy			
Demand at T = Low	0.8801	0.1199	0.0000
Demand at T = Normal	0.3309	0.6469	0.0222
Demand at T = High	0.0000	0.1116	0.8884
Price on Electric Energy (Normal Distributions, Mean, Normalized)			
Supply =	Low	Normal	High
Demand at T = Low	0.8794	1. 0344	1.3702
Demand at T = Normal	1.0027	0.9054	0.9188
Demand at T = High	1.1951	1.2513	1.3896
Price on Electric Energy (Variance, Normalized)			
Supply =	Low	Normal	High
Demand at T = Low	0.0341	0.0616	0.0100
Demand at T = Normal	0.0342	0.0815	0.0100
Demand at T = High	0.0680	0.0863	0.0275

The FHMM provides more insight into electricity markets such as the interaction of demand on electric energy and ancillary services, the interdependency between demand on electricity and supplied online generation capacity. Although the FHMM is much richer than regime switching example, it is still a very simplified model. More complicated HMM could provide the foundation to launch the Monte Carlo simulation, which could be enhanced to incorporate more structures. The FHMM estimated assumes no transmission congestion, which is justified the strong transmission network at "West" control area of NYISO.

More work on modeling such as incorporating the impacts of transmission networks in details requires more information about the specified market than is readily available, and will be discussed in section 3.4.

3.4 Location Marginal Price- Zonal/Nodal Pricing Framework

The HMM discussed in section 3.3 implicitly assumes an integrated electricity market supported by a perfect transmission network. However, the current electric power transmission network was not designed to support commerce. Congestions of electric power transmission network segment into electricity markets into zones and buses. Location Marginal Price- Zonal/Nodal Pricing Framework trades electric energy and other services according to their time-varying and location dependent values [51]. Location Marginal Price (LMP) has seen more and more implementations for its recognition of the time varying and location dependent value of electricity.

The LMP for a specific electric power system bus at time T is defined as the marginal cost of serving electric energy if a small incremental of load is to be served. The LMP for any electric power system bus/node is jointly determined by the offers provided by Generation Companies (GENCO), bids provided by Electric Service Companies (ESCO) or Load Serving Entities (LSEs), and the status of transmission network. Figure 20 shows the LMP framework implemented in PJM [21].

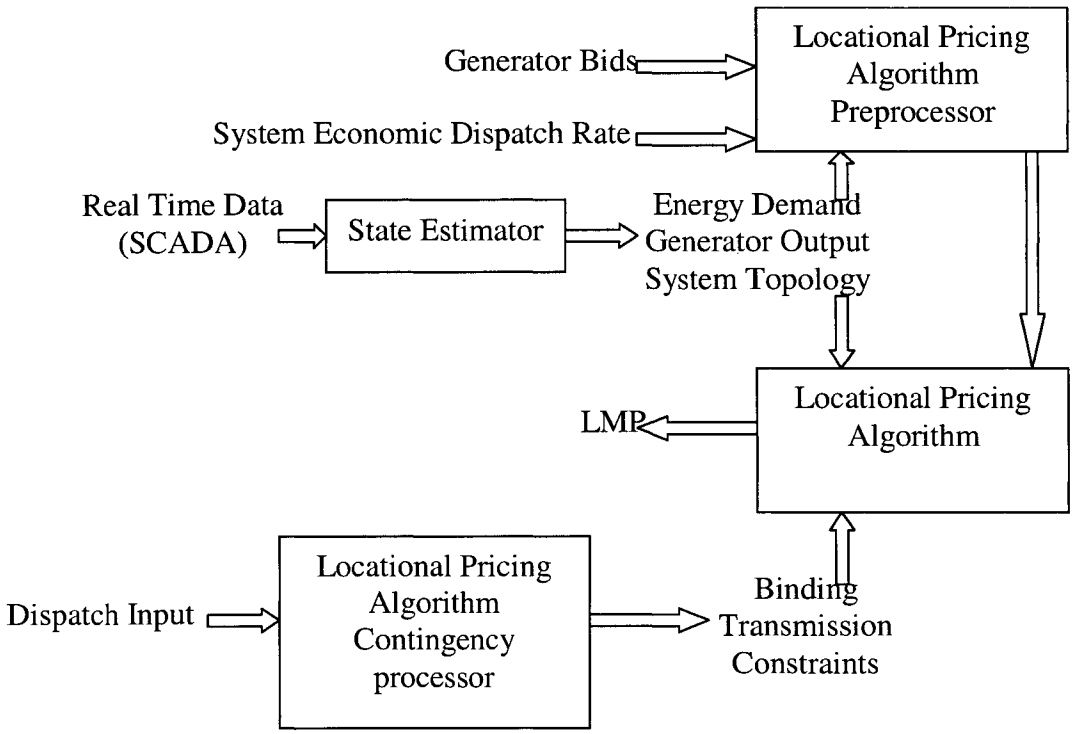


Figure 20: LMP Framework Implemented in PJM

The pricing algorithm shown in Figure 20 captures both the locational nature and the marginal nature of LMP. The locational nature of LMP is determined by the topology of the electric power transmission system. A contribution vector is defined to represent the electric power transmission system physical driver in Equation 9. The marginal nature of LMP is captured by the incremental cost of supplying one more unit of electric energy. An offering price vector is defined to describe the economic drivers, namely the offers provided by Generation Companies (GENCO), in Equation 9. The physical and economic drivers jointly determine LMP, which is the product of the contribution vector and the offering price vector shown in Equation 9.

$$LMP\_Bus_i = \begin{bmatrix} ContributionFactor_1 \\ \vdots \\ ContributionFactor_n \end{bmatrix} * [GENCO\_Offer_1 \quad \ldots \quad GENCO\_Offer_n]$$

$$subject\ to: \sum_{i=1}^{n} ContributinFactor_i = 1$$

$$n: Numbers\ of\ Buses\ connected\ with\ Bus_i$$

Equation 9: LMP Decomposition as Physical and Economic Drivers

An example clarifies the concept and algorithm of LMP. Figure 21 illustrates a simplified fully connected three-nodes transmission network. All three transmission lines are assumed to have equal reactance, which means that the electric distances between three buses are the same. The topology and electric distances between buses jointly determine how electric energy flows through the transmission network, namely the feasible set of contribution factors. Each transmission line is assumed to be capable of transferring 100MW of electric energy.

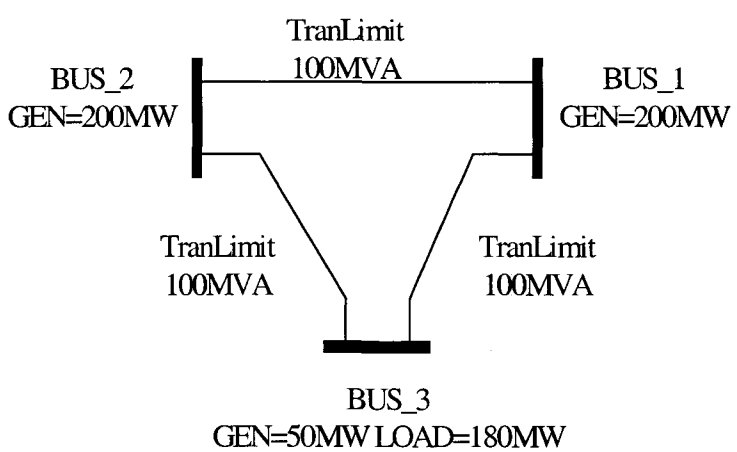


Figure 21: Illustration of LMP

The load is assumed to have no self-price elasticity, which holds true in current electricity markets. Generation resources are assumed to have different but constant

production costs to simplify this example. Table 5 illustrates the generators' offers, generation outputs, and the final LMP at all three buses.

Table 5: LMP Example Calculation

	Bus_1	BUS_2	Bus_3
Generation Offers	$10/MWh	$12/MWh	$20/MWh
Generation Output	120MW	60MW	0MW
Load	0MW	0MW	180MW
LMP	$10/MWh	$12/MWh	$14/MWh

It is assumed that the load at BUS_3 is to increase by 1MW, and then the generation output from BUS_1 needs to decrease by 1MW while the generation output from BUS_2 needs to increase by 2MW. The exact numbers are determined by the physical parameters of electric power transmission system and physical laws. In this example, it is the topology of the three buses transmission system and equivalence of transmission line reactance. The total increase cost of serving the load of one more MW at BUS_3, the LMP at BUS_3, is calculated as in Equation 10.

$$\begin{aligned} LMP\_BUS_3 &= [\$10/MWh, \$12/MWh]\begin{bmatrix} -1 \\ +2 \end{bmatrix} \\ &= \$14/MWh \end{aligned}$$

Equation 10: LMP Example Calculation

As shown in this example, the contribution factors of transmission system are defined by the topology of transmission system, demand to be served, and Available Transmission Capability (ATC). The offers entering LMP calculation are defined by merchant generators' offers.

3.5 Modeling LMP with HMM

LMP captures not only the demand and supply forces of electricity but also the operational impacts of the electric power systems. The modeling of LMP requires modeling of both the underlying physical electric power system and modeling of the economic supply and demand forces. This section proposes to model the movement of LMP with HMM. The movement of LMP is decomposed into physical and economic drivers as in HMM.

The physical drivers include transmission network topology and generation technology. The physical electric power system defines the technologies of the electricity markets. The technologies of electricity markets define the cost structures of electricity markets, including both the costs of electricity generation and transmission. The electricity cost structure of electricity provides a foundation for modeling the market prices of electricity for an efficient electricity market sets the market prices of electricity to the marginal costs of supplying electricity. The marginal cost of supplying electricity includes generation cost, transmission cost and losses.

The marginal costs of generating electricity are determined by the incremental cost of the marginal generator, which is in turn jointly determined by demand level, cost of fuel, generation composition and environmental cost. The generation compositions of electricity markets, also referred to as generation stack, differ based on the availabilities of raw energy resources, environmental regulations, and transportation supports. Figure 22 categorizes the sources of raw energy sources into two groups: Fuel traded on Markets and Renewable energy not tradable.

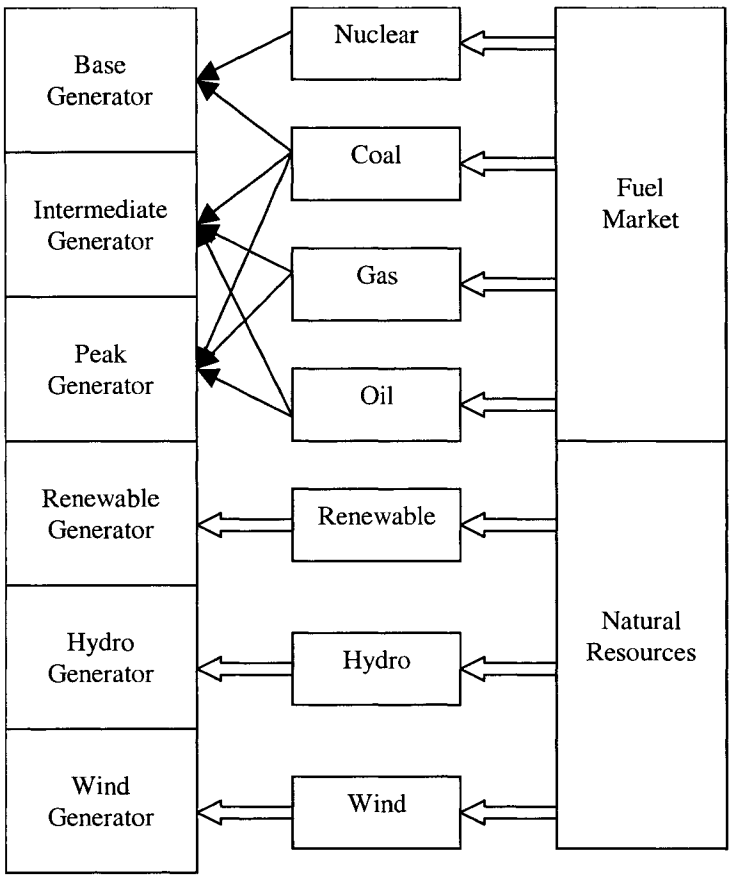


Figure 22: Energy Sources

Table 6 illustrates the electricity generation composition of several regional electricity markets in North America. It is shown that the generation composition could be significantly different between electricity markets.

Table 6: Generation Technology Composition in U.S. Electricity Markets

	PJM	NERTO	NYISO	ERCOT	CA
Gas	31%	32%	90%	75%	46%
Oil	21%	28%	74%	38%	1%
Coal	37%	9%	0%	21%	0%
Nuclear	21%	16%	0%	7%	9%
Hydro	5%	12%	1%	1%	23%
Others	1%	2%	0%	0%	21%

Some generator could switch fuels, thus the sums of percentages do not necessarily equal to 100%.

The marginal cost and market values of transmission service diverge on electricity markets. The marginal cost of transmitting electricity is either zero or infinity depending on the status of a specific transmission system. When extra transmission capacity is available, the marginal cost of transmitting electricity is zero. The marginal cost of transmitting electricity is infinity when transmission congestions occur. Dispatching more expansive generation sources or withholding demand on electricity could mitigate the lack of transmission capability. This provides synthetic transmission capability, and leads to market values of transmission capability. The market values of transmission services are captured by the differences of LMPs at different locations on transmission systems.

The physical nature of electric power system determines not only the cost structure of supplying electricity; it also has significant impacts on the movement of LMP. The non-storable nature of electricity contributes to the significant volatilities and spikes of LMP. Another important driving force of the electricity market comes from economic drivers.

The economics drivers determine the structure and architecture of electricity markets. The structure and architecture of electricity markets define how electricity market fluctuates around its equilibriums and how LMP differs from marginal costs of supplying electricity.

The notion of market structure, developed as part of the "structure-conduct-performance" paradigm of industrial organization, measures the concentration for the ownership of production capacity with indexes such as Hirschmann Herfindahl Index (HHI). Although the electric power industry restruction breaks the vertically integrated electric utility structure into horizontally independent entities market structure, electricity markets are not perfectly competitive. Imperfect electricity markets observe the divergence of market prices and marginal cost of supplying electricity.

Market architecture defines how market players interact with each other and how electricity and information are exchanged and shared on electricity markets. Market architectures are implemented as a set of sub-markets and the linkages between them. The sub-markets could be categorized by many criteria such as commodity traded, contracts traded, the trading mechanism and authority of ISO/RTO. The linkages between sub-markets may be implicit price bindings enforced by arbitrage or explicit rules linking rights purchased

in one market to activity in another. The market architecture provides incomplete market information to market participants, whose profit-maximization behaviors jointly determine the movement of electricity markets.

The economic drivers also include fuel prices, demand uncertainties, and profit maximization of market participants with incomplete information. The economic drivers define how LMP differs from cost while the physical drivers of electricity markets defines the cost structure of electricity market. System-states-depending random processes are utilized to capture the impacts of economics drivers.

An example illustrates to decompose LMP movements into physical and economic drivers as a FHMM. LMPs are modeled to be weighted sums of samples from the GENCOS' offer distributions. The GENCOS' offers are modeled with probabilistic distributions. The contribution factor determines the weights assigned to GENCOS' offers. The dynamics of contribution factors are defined with partially observable Markov processes as shown in Figure 23.

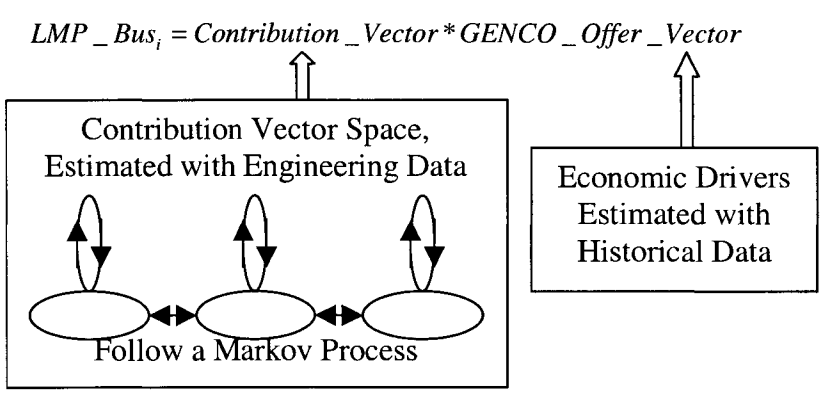


Figure 23: Modeling LMP with FHMM

The contribution factors capture the impact of transmission system and could be estimated using engineering data on electric power systems. Multiple contribution factors compose a closed system state space. Markov processes are employed to capture the transition of contribution factors within the system state space. A state transition probability matrix describes the probability of contribution factor change from one state to another, conditional on the demand level of electric power and other factors.

System-dependent GENCOs offers' distributions capture the generation composition, fuel cost, demand, and most importantly market dynamics. The distributions could be calibrated using historical market data. The marginal GENCOs offers could be further decomposed into Selection Factor, "financially synthetic electricity cost", and the cost-offer spread. The selection factor identifies marginal generators from the generation stack. The financially synthetic electricity cost captures the heat rate and fuel cost. The cost-offer spread captures the profit-maximization behaviors of GENCOS. Equation 11 illustrates the further decomposition of GENCOs' offers.

$$
\begin{aligned}
GENCO\_Offer_i = & \begin{bmatrix} SelectionFactor_1 \\ \vdots \\ SelectionFactor_m \end{bmatrix} \\
& * \left[HeatRate_1 * Fuel_1 \quad \ldots \quad HeatRate_m * Fuel_m \right] \\
& + \; Cost\_Offer\_Spread \\
subject\ to: \quad & SelectionFactor \in \{0,1\} \\
& m: \; Numbers\ of\ Generation\ Units\ at\ Bus_i
\end{aligned}
$$

Equation 11: Further Decomposition of GENCOs' Offers

Combined together, this approach models LMP to be generated from multiple random processes. The proposed hybrid model is mathematically equivalent with structured regression models. The dependent variable LMP is modeled to be dependent on a set of independent variables such as fuel prices, load levels and others. The FHMM distinguishes itself with simple regression models by assuming structured multiple regression equations. This is similar with the regime-switching model, which assumes that electricity markets to transition between multiple regimes. However, the regime-switching model relies only on historical market data, therefore does not incorporate physical drivers of electricity markets. FHMM combines the strengths of both fundamental economic modeling and mark-to-market stochastic modeling. The physical driver captures the strengths of engineering-based production cost modeling approach while the economic drivers captures the strength of mark-to-market approach. The decomposition provides a modeler the capability and tools for

structural modeling of LMP. The key market drivers could be identified and be appropriately modeled to capture the heterogeneous nature of electricity.

An example is given to illustrate the decomposition of LMP with FHMM. The following example models the LMP at a 500KV bus in PJM with the proposed FHMM. A portion of the 500KV transmission network in PJM is simplified to a fully connected three-buses transmission network and shown in Figure 24.

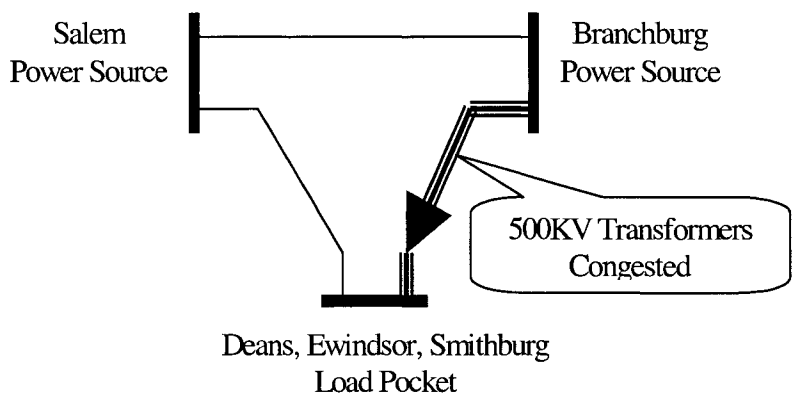


Figure 24: Simplified PJM 500KV Transmission Network

This example focuses on LMP at a 500KV bus, DEANS. Four other 500KV buses, BRANCHBURG, EWINDSOR, SALEM, and SMITHBURG, connect to DEANS. The correlations between hourly LMPs at DEAN, EWINDSOR and SMITHBURG are very high, thus modeled as one single bus as load pocket. Both of SALEM and BRANCHBURG are modeled as generation sources. SALEM is connected to a nuclear power plant while BRANCHBURG is connected to a few power plants. Bus SALEM and BRANCHBURG are modeled to be connected in order to capture the impact of remaining 500KV transmission network. The historical price data used are from the Real Time market of PJM from June to Aug 2004. Weekends and holidays are removed, which leads to 65 working days with 1560 samples. This example focuses on decomposing physical drivers and economic drivers of LMP by adopting simplified models for both transmission network and economic drivers.

The modeled congested transmission route is the 500KV transmission line from BRANCHBURG to DEANS. The contribution factors are estimated using historical data. A linear regression on the LMPs for congested hours shows that:

$$LMP\_Deans = [LMP\_Branchburg, LMP\_Salem] * [\beta_1, \beta_2]$$
$$[\hat{\beta}_1, \hat{\beta}_2] = [-0.4289, 1.3247]$$
$$95\%\ Confidence\ Interval\ for\ [\beta_1, \beta_2] = \begin{bmatrix} -0.5037 & 1.2890 \\ -0.3541 & 1.3603 \end{bmatrix}$$

Equation 12: LMP Contribution Factors

Although the $\sum_{i=1}^{n} ContributinFactor_i = 1$ constraint is not strictly enforced, solving a constrained regression could easily satisfy it. The residual plot illustrated in Figure 25 demonstrates a satisfactory fit except less than 10 outfitters. A stress-testing model should capture such extreme events.

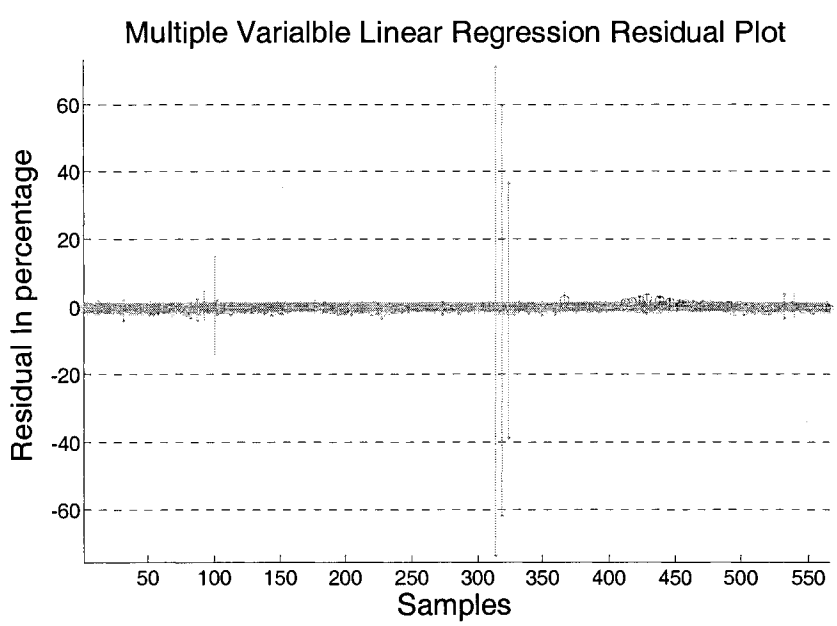


Figure 25: Contribution Factor Estimation Residual Plot

The transmission system transition probability matrix is estimated using historical PJM transmission network operating data

$$\begin{bmatrix} Transition\_Matrix & No-Congestion & Congestion \\ No-Congestion & 0.943478 & 0.056522 \\ Congestion & 0.158537 & 0.841463 \end{bmatrix}$$

Equation 13: Markov Transition Probability Matrix

The hypotheses that LMPs at BRANCHBURG and SALEM during congestion follow normal/lognormal distributions are significantly rejected. This work models normalized LMPs during congestion to follow beta distributions.

$$\begin{aligned} &Branchberg \in Beta(2.7551, 2.7079) \\ &95\%\ Confidence\ Level = \begin{bmatrix} 2.5921 & 2.5247 \\ 2.9182 & 2.8911 \end{bmatrix} \\ &Salem \in Beta(6.6749, 3.1181) \\ &95\%\ Confidence\ Level = \begin{bmatrix} 6.4713 & 2.9488 \\ 6.8785 & 3.2875 \end{bmatrix} \\ &Deans \in Beta(2.0597, 3.3121) \\ &95\%\ Confidence\ Level = \begin{bmatrix} 1.9126 & 3.1425 \\ 2.2027 & 3.4888 \end{bmatrix} \end{aligned}$$

Equation 14: GENCOs' Offer Distribution

As shown in this example, this hybrid model provides valuable insight and flexibility into factorizing the uncertainties observed in LMP movements. Such capabilities are of importance since significant risks and opportunities confront GENCOs often during congested peaking time intervals.

The advantages of FHMM over the time series approach lie mainly with the interpretation of volatilities and correlations. The significant volatilities and unique volatility patterns are interpreted by realizing that all the observed samples are drawn from multiple distributions. This feature also helps to interpret the correlation between LMP and fuel

prices. The prices spikes observed are explained as the outputs of low-probability states. The clusters of prices spikes are interpreted by the instabilities of low-probability states.

The advantages of HMM over the production cost modeling approach lies mainly with the utilization of history market data and capturing of economic drivers. The assumptions that GENCOs bid at their marginal cost are removed by incorporating historical market data. The replacing of detailed modeling of electric power transmission system with a Hidden Markov processed for contribution factors significantly improves the computational efficiency, reduces the modeling efforts, and most importantly changes the deterministic results into stochastic results.

3.6 Extended HMM into Mid/Long-Term Electricity Markets Modeling

HMM can also be used to model electricity markets in mid-term and long-term. Mid-term market movements are assumed to be limited to changes caused by the different competition strategies employed by market participants within the same market structure. The same HMM with different sets of parameters is used to model the mid-term market movement as shown in Figure 26. An electricity market is assumed to settle with different equilibrium within the same market structure until a threshold is reached, which in turn leads to changes of market structure in the long term.

Long-term market movements are assumed to be market structure changes such as generation units' entries and retirements, long-term demand trends, fuel costs, transmission networks, and technology advancements. HMMs with different structures are employed to model the long-term market movements as shown in Figure 26

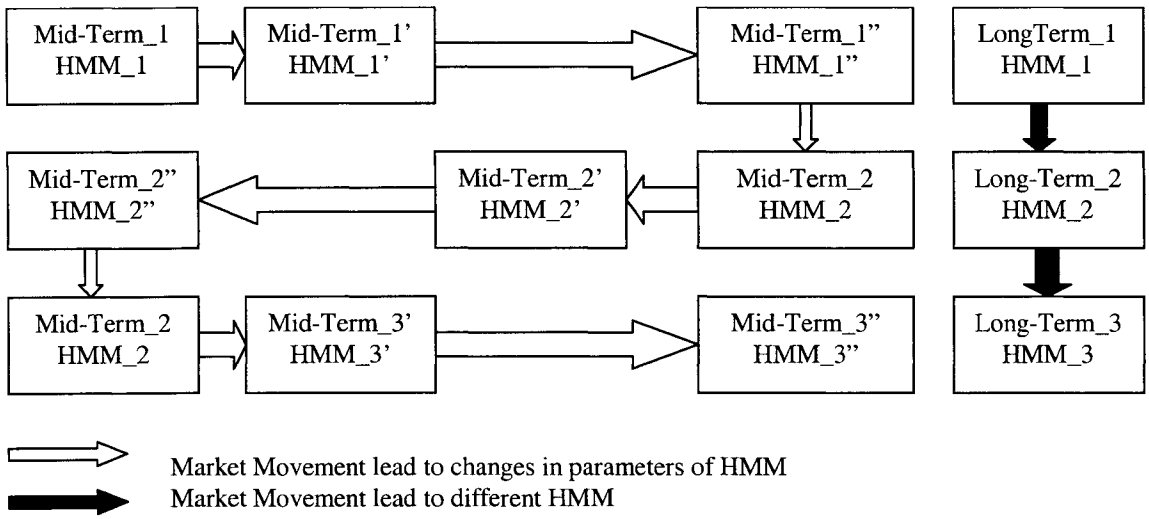


Figure 26: Mid/Long-Term Electricity Market Modeling with HMM

High-quality forecasts for Mid-/Long-Term electricity markets are prohibitive expensive if all possible. This work proposes to capture the extreme uncertainties and fuzziness with fuzzy sets and numbers. The fuzzy approach allows a modeler to incorporate subjective judgments from experts. Most often, such estimation is of fuzzy nature and could not be fitted into a probabilistic way without sacrificing it values.

Fuzzy numbers could enter HMM by many ways. This work proposes construing a fuzzy states transition matrix to capture the fuzziness of electricity market movements. Fuzzy numbers are employed to define the possibilities of electricity market transition. Fuzzy numbers are special cases of fuzzy sets and this work applies triangle fuzzy numbers as shown in Figure 27. A symmetric triangle fuzzy numbers is defined by its center α and half-width c.

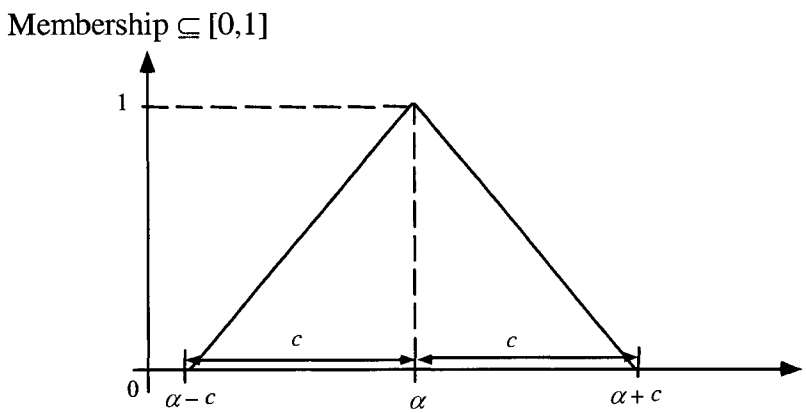


Figure 27: Symmetric Triangle Fuzzy Number

A fuzzy function is a generalization of a classical crisp function and it can be defined in many ways. This work adopts a classical definition for fuzzy function of fuzzy numbers as illustrated below.

Assume a function $Y = f(X, A)$ with its inputs, outputs and parameters $X, Y, and\ A$ all defined as crisp numbers. Given a parameter set A, this function maps X to Y. Given a fuzzy number $\tilde{A}$, a fuzzy function $f : X \rightarrow \varphi(Y)$ mapping X to $\varphi(Y) : set\ of\ all\ fuzzy\ subsets\ on\ Y$ is defined by a membership function on Y.

$$\tilde{Y} = f(x, \tilde{A}),$$

Membership function for $\tilde{Y}$

$$\mu_Y(y) = \begin{cases} \max\limits_{\{a|y=f(x,a)\}} [\mu_A(a)], & \{a \mid y = f(x,a)\} \neq \varnothing \\ 0, & otherwise \end{cases}$$

Equation 15: Definition of Fuzzy Function

This chapter models electricity markets with Hidden Markov Model. This modeling approach is the foundation of an integrated generation assets valuation solution.

Chapter 4: Valuation of Generation Assets

This chapter focuses on the valuation of generation assets. Generation assets are modeled as real options to capture the operating flexibility embedded into generation assets. Section 4.1 introduces the approach of real option and focuses on the formulation and solution of real option problems. Section 4.2 valuates generation assets as real options. The valuation problem is formulated as Partially Observable Market Decision Problem (POMDP) and solved with Markov Lattices. Section 4.2 also estimates values of information by comparing the difference between POMDP and MDP. Section 4.3 focuses on investment opportunities in generation assets and incorporates fuzzy information.

4.1 Real Option as a Valuation Approach

Real option theory extends financial option theory to real assets, and incorporates a few features associated only with real assets. The formulation and solution of real option problem should be capable to incorporate the features of real option.

The first feature is market movements. The underlying assets of real options are often consumable commodities, which not only have investment values but also satisfy consumption needs. The prices of commodities behave differently with financial assets. Commodities prices often demonstrate seasonality, mean-reversion and other features. This feature needs to be addressed during the modeling process of the underlying markets. Chapter three proposes to capture this feature using HMM.

Secondly, the alternatives for exercising/operating real options are different. There are physical constraints on the exercises of real options. While most financial options can be traded/exercised freely as long as markets are open, the exercise of real option is subject to operating constraints. The timing constraints of generation assets significantly reduce the available alternatives to GENCOs. The set of alternatives for operating real assets are more limited. There are often non-trial costs associated with exercising real option, which further reduces the economic feasibility of alternatives. The startup and shutdown of generation

assets could be expensive under some situations that GENCOs might be better off if running at loss. Also, an owner of real options can change the value of its real assets by actively managing its real assets. The owners of financial options only have the choice to trade, hold or exercise the options while the owners of real assets could choose between upgrading, mothballing among other choices. In fact, more active management can be done on real options, compared to passively trading/exercising on financial options. This feature needs to be addressed from the perspective of solving a real option problem. This chapter begins with a review of the past research on formulating and solving generation assets real options.

Methods to valuate real options are based on methods to valuate financial options, and modified to incorporate features of real assets. The following section categorizes approaches by their information requirement and techniques to analyze information.

The prices movements of electricity could be defined by partial differential equations and solved with close-formed formulas. Partial differential equations need the least information during the modeling stage, and provide solutions similar to Black-Scholes' formula for financial options. The tradeoffs are strong assumptions on markets, price movements and loss of flexibility. Often, this approach assumes that:

- Forward markets for both of electricity and fuel are complete and efficient in the sense of arbitrage free
- Ramp ups and ramp downs of the plant can be done immediately, thus no physical constraints
- Prices of electricity follow a special stochastic process and distributions such as GBM and Log-normal distributions

Fleten et al. [53] investigate optimal entry and exit threshold values of spark spread for gas fired power plant using close-form approach. The solution provides more insights than traditional discounted cash flow approaches. Uncertainty increases the threshold to build a new power plant while decreases the threshold to abandon an exiting power plant.

Differential equations provide a rough estimate of values of generation assets at a very low cost of information. However, a lot of features are lost and its application is very limited. Close-form approaches have difficulties on valuating American, Asian and exotic options.

Improvements and refinements on this approach have been under intensive research. Features such as price spikes, mean-reversion, and others are added into models for electricity markets movements

Multinomial tree is a more flexible technique to valuate real options, for it is easy to implement and understand. One advantage of multinomial tree is its capability to valuate American option, which can be exercised earlier before its maturity. Another advantage is the capability to incorporate carrying cost, which could be maintenance cost for idling a power plant.

Multinomial tree approximates the operation of generation assets by discrete production levels, and is very similar with stochastic dynamic programming. Both of them utilize backward induction, and optimize the operation of generation assets. Gardner et al [16] applied stochastic dynamic programming to valuate generation assets, and mean-reversion GBM for price movement is assumed. Their research focused on searching of optimal operating policy boundaries, which are defined as expected spark spread at a future time accounting for time constraints for a generator to reach that optimal state. Deng et al [14] used quadrinomial tree to implement the spark option assuming that the prices of fuel and electricity are correlated. They also investigated the impact of physical constraints under different price movements, and concluded that the mean-reversion price movement enhances the impacts of physical constraints. Also, they pointed out that GBM does not present electric power prices well.

Multinomial tree or stochastic dynamic programming makes compromise between accuracy of modeling and computing requirement. Further refinement of the modeling of price movement would increase the accuracy of valuation and also makes the model more complex, for example seasonality of prices on electricity addressing peak and off-peak price movements. However, stochastic dynamic programming is not an open framework for it is hard to add more than one commodity and other operation objectives because it suffers the dimensional curse. Most of the researches deploying multinomial tree or stochastic dynamic programming assumed only electric energy, and ignored ancillary services. In fact, the values of generation assets depend closely on prices of both electricity and fuels.

Another approach for solving real option is simulation. Tseng et al [17] integrated forward-moving Monte Carlo simulation with backward-moving dynamic programming to investigate the impact of physical constraints on valuation of generation assets. The research focused on searching for indifference locus on which the unit commitment decisions "on" and "off" are equivalent in terms of expected profit. The authors pointed out, the iterative forward simulation and backward induction requires massive computations. They also concluded that simulation provides extra flexibilities, such as irregular price processes or other commodities like spinning reserves.

Simulation utilizes brute-force to valuate generation assets, while it still needs to assume distributions and price movement. Massive computation limits the application of simulation to mid-term and long-term valuation. Computation burden also limits the sensitivity analysis, which could be easily provided by close-formed approaches. Sensitivity analysis is of important in the operation of GENCOs, providing guideline for hedging and risk management.

Past researches assumed static markets with static market structure and architectures. Mid-term and long-term valuations are achieved by multiplying short-term results.

Table 7 summarizes the three approaches to apply real option analysis to valuation of generation assets. More discussion on real option analysis and its application can be found in references [54].

Table 7: Comparison of Real Option Solution

	Differential Equations	Multinomial Tree	Simulation
European/American	Yes/No	Yes/Yes	Yes/Yes
Flexibility	No	Medium	Max
Computation Power	Least	Medium	Max
Price Movement	GBM/Spikes	GBM	GBM/Other
Physical Constraints	No	Yes	Yes
Market Structure	Liquid	Liquid	N/A
Commodities	Single	Single	Multiple

4.2 Formulation of Generation Assets with MDP and POMDP

This section identifies Markov Decision Problem (MDP) and Partially Observable Markov Decision Problem (POMDP) as real option analysis approaches. The operation of generation assets is first formulated as an MDP with full observability, and then extends to POMDP with partial observability.

4.2.1 Formulation of Generation Assets Valuation as MDP

MDP has seen successful application with sequential decision-making because it optimizes between the immediate rewards and the future gains to yield the optimal solution. When the electricity markets are modeled with Markov Processes, it is shown that the operation of generation assets can be modeled and solved with MDP. Figure 28 demonstrates the application of MDP to the valuation and operation of generation assets. Linear programming, dynamic programming, and other algorithms can be used to solve an MDP [37]. MDP not only provides the value of generation assets, but also the optimal operating policy.

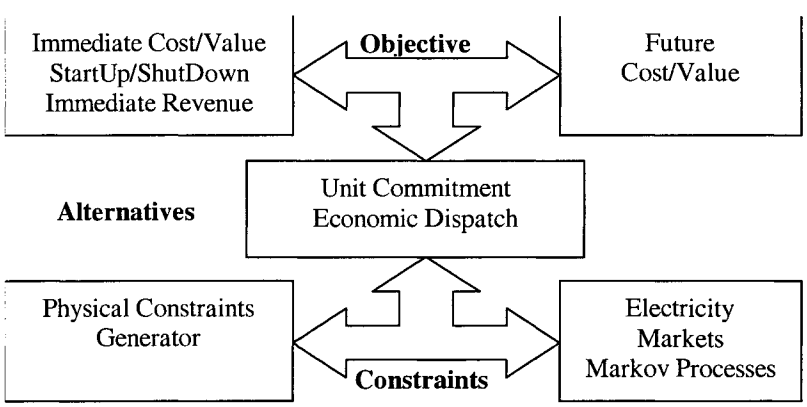


Figure 28: MDP for Generation Assets Valuation

A decision maker using MDP is in fact following a real option analysis approach in the following way. First, MDP recognizes and incorporates the operation flexibilities by allowing decision makers to respond differently based on the latest observations. Second,

MDP measures the option value using expected future gains. MDP often finds optimal strategies, where foregoing immediate rewards captures the future option values. The relationship between immediate rewards and expected future gains is similar to the relationship between option intrinsic value and option time value.

The sequential decision problem solved by MDP is identical to a real option problem where uncertainties are structured to evolve as Markov processes. Figure 29 defines a Markov lattice to model the evolvement of electricity markets states, where electricity price movement at each state is approximated with a multinomial tree. Markov lattices have been shown to be easier to construct and converge faster than multiple-period multinomial trees when applied to option pricing [38]. The probability embedded in the Markov processes should be replaced with risk-neutral probabilities if a single risk-neutral discount rate is to be used. The risk-neutral probabilities do not equal the true physical probabilities but are modified to construct a risk-neutral world, where a single risk-free discount rate is used throughout the decision process. Hull states that it could be assumed that electricity price behaves in the same way in risk-neutral and real world. The reason is that the percentage change in electricity price has negligible correlation with market return. Therefore, the risk-neutral growth rate on electricity could be estimated by real world observation. Modeling generation assets as real option on electricity could use the risk-neutral growth rate as the growth rate and no adjustment is need [7].

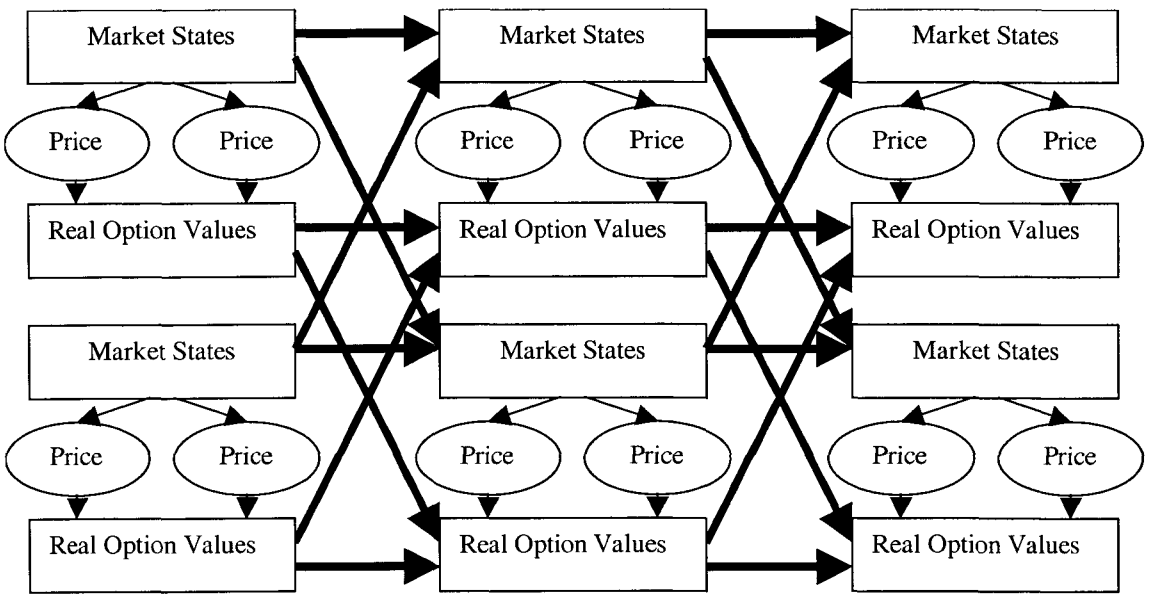


Figure 29: Markov Lattice based on HMM

The following example illustrates the application of MDP to formulate and solve generation assets valuation. Assume an electricity market, which is defined as a FHMM shown in Figure 30. The prices on electric energy and ancillary service are determined simultaneously by supplied online generation capacities and demand level. Demand on electric energy is assumed to evolve autonomously according to a Markov chain. The demand on ancillary services is assumed to be dependent on energy demand. The online generation capacity is modeled to follow another Markov process, depending on the previous supply level and energy demand.

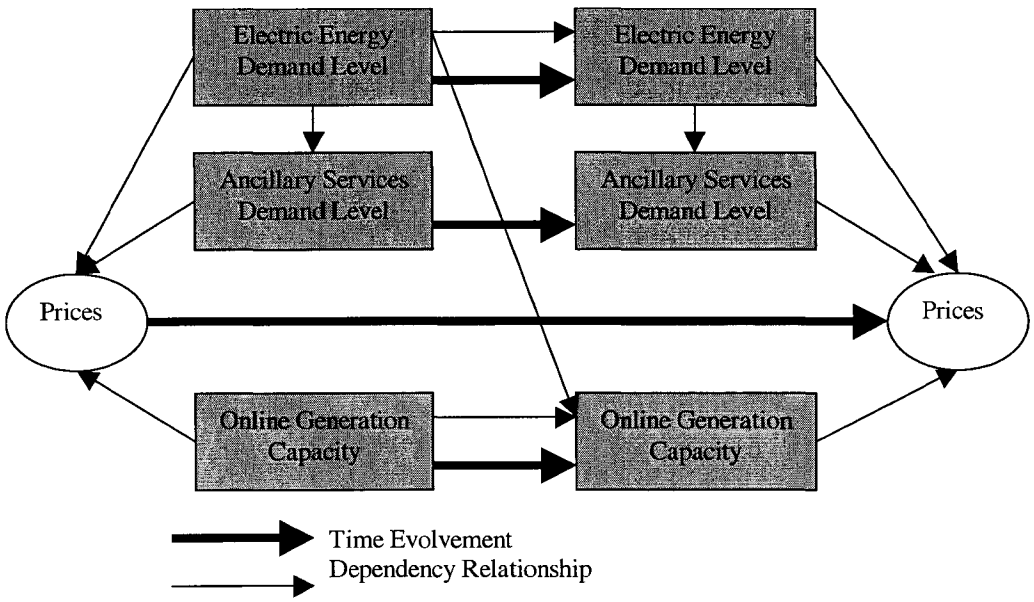


Figure 30: Electricity Market Movements

A thermal generation unit is valued using a Markov chain model in this example. The physical constraints for the given generation unit is shown in Table 8. The demand on electricity and supplied online generation capacity are modeled to have three states including low, intermediate and high. The transition of demand on electricity is modeled to follow a Markov chain defined in Table 9. The transition of supplied online generation capacity is assumed to be dependent on the previous level of demand and capacity supply. The interdependency is modeled to be following a Markov chain defined in Table 10. Only one Markov chain is utilized for the whole time span for simplicity. More Markov chains and system states could be used to increase the accuracy at cost of computational burden. The prices on electric energy and ancillary services are assumed to follow normal distributions shown in Table 11 and Table 12. Prices in Table 11 and Table 12 are normalized according to the daily price pattern shown in Table 13. Equation 16 provides mathematical definition for Table used. All data are generated by market simulations.

$$Table\_9: Probability(Demand_{t+1} = State_i \mid Demand_t = State_j)$$
$$Table\_10: Probability(Supply_{t+1} = State_i \mid Supply_t = State_j \cap Demand_t = State_k)$$
$$Table\_11: Mean\ of\ Price\ at\ Time\ t\ if\ Supply_t = State_j \cap Demand_t = State_k$$
$$Table\_12: Variance\ of\ Price\ at\ Time\ t\ if\ Supply_t = State_j \cap Demand_t = State_k$$
$$Where\ i, j, k \in 1..N, N: Numbers\ of\ States$$

Equation 16: Definitions for MDP Valuation Tables

Table 8: Characteristics of Generation Asset

Generator Physical Characteristics	
Minimum Up Time, Ton	1 Hours
Minimum Down Time, Toff	1 Hours
Start-Up Time, Tup	1 Hours
Shut-Down Time, Tdown	1 Hours
Ramp Rate, Ramp	100MW/Hour
Minimum Output Level, Qmin	100MW
Maximum Output Level, Qmax	400MW
Heat Rate: H(p), P in MW Approximated by Piecewise linear functions later	78+7.97*P+0.00482*P^2 MMbtu/MWh

Table 9: Demand Transition Probability Matrix

Demand	Low at T+1	Mid at T+1	High at T+1
Low at T	0.9229	0.0771	0.0000
Mid at T	0.0199	0.9634	0.0168
High at T	0.0000	0.1085	0.8915

Table 10: Supply Transition Probability Matrix

(Low Supply at T)			
	Supply Low at T+1	Supply Mid at T+1	Supply High at T+1
Demand Low at T	0.9585	0.0315	0.0100
Demand Mid at T	0.9434	0.0466	0.0100
Demand High at T	0.0000	1.0000	0.0000
(Matched Supply at T)			
	Supply Low at T+1	Supply Mid at T+1	Supply High at T+1
Demand Low at T	0.2097	0.7803	0.0100
Demand Mid at T	0.1531	0.8369	0.0100
Demand High at T	0.5898	0.4002	0.0100
(Oversupply at T)			
	Supply Low at T+1	Supply Mid at T+1	Supply High at T+1
Demand Low at T	0.0000	0.9900	0.0100
Demand Mid at T	0.0000	0.9900	0.0100
Demand High at T	0.0000	0.9900	0.0100

Table 11: Mean of Prices on Electricity

	Supply Low	Supply Mid	Supply High
Demand Low	0.8794	1. 0344	1.3702
Demand Mid	1.0027	0.9054	0.9188
Demand High	1.1951	1.2513	1.3896

Table 12: Variances of Prices on Electricity

	Supply Low	Supply Mid	Supply High
Demand Low	0.0341	0.0616	0.0100
Demand Mid	0.0342	0.0815	0.0100
Demand High	0.0680	0.0863	0.0275

Table 13: Daily Pattern of Prices on Electricity

Daily Price Pattern								
Time (Hour)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Price ($/MWh)	15	15	15	15	17	19	21	23
Time (Hour)	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Price ($/MWh)	25	27	30	34	40	50	40	34
Time (Hour)	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Price ($/MWh)	30	27	25	23	21	19	17	15

The optimal operating policy of the investigated generation unit defines the optimal action for each state. Past researchers searched for a "spark spread" threshold. A generation unit is turned on when the expected "spark spread" is greater than the threshold. This example shows that the exercises of real option, self unit commitment decision, should not only depend on price levels, but more importantly on the current market states of demand and supplied online generation capacity. It is shown that a generation unit should not be turned on when the demand and supply is in balance even a relative high price is observed. The reason is that a relative high price could occur due to randomness in market but not demand and supply forces. It is observed that the investigated generation unit should only be turned on when demand is high and the market is not in balance. A decision based on both underlying economic forces and observed prices provide more value to GENCOs.

Table 14 defines the optimal generation unit operating policy for the specified electricity market. The up-left corner of optimal self unit commitment policy defines a price, under which the investigated generation unit should stay off. It is observed that a Generator needs lower prices to turn on when the supply is at low. Much higher prices are needed for the investigated generator to turn on when the market is already over-supplied.

Table 14: Optimal Self Unit Commitment Policy

Policy 1 (Low Supply)			
	Demand Low	Demand Mid	Demand High
Price	31.87	36.35	44.81
Decision	Turn On	Turn On	Turn On
Price	29.77	33.94	39.11
Decision	Turn Off	Turn Off	Turn Off
Policy 2 (Matched Supply)			
	Demand Low	Demand Mid	Demand High
Price	38.54	34.41	47.78
Decision	Turn On	Turn On	Stay Off
Price	34.07	29.23	40.21
Decision	Turn Off	Turn Off	Turn Off
Policy 3 (Oversupply)			
	Demand Low	Demand Mid	Demand High
Price	48.48	32.51	49.94
Decision	Turn On	Turn On	Turn On
Price	47.52	31.87	47.27
Decision	Turn Off	Turn Off	Turn Off

Table 15: Generation Assets Values v.s. Price Volatilities

	Half Variance	Benchmark	Double Variance
Generation Unit Value	0.9647	1.000	1.1434

Table 16: Generation Assets Values v.s. Volatilities of Demand and Supply

	More Volatile Demand	Benchmark	More Volatile Supply
Generation Unit Value	1. 2532	1.000	1.1826

For a one-week valuation, Table 15 and Table 16 give the normalized value of a thermal generation unit with an initial state as off at 1AM under different scenarios. The focus is on the comparison of the impacts of changes on demand and supply, and changes on the bidding strategies of market players. The changes of demand and supply are based on economics forces such as demand composition, fuel prices, and new generation technologies. These changes are defined by the demand and supply transition probabilities matrix. Bidding strategy changes are defined by the distributions of prices for each market state. Different distributions result in different volatilities and payoffs at each market states. All values are normalized to facilitate comparison.

As demonstrated in this example, MDP is identified as a real option approach, which enables the dynamic real-time valuation of assets as new information on market state is

received. It is also shown that optimal exercise of real option depends on the state of electricity markets, but not the observed markets prices.

4.2.2 Formulation of Generation Assets Valuation as POMDP

The MDP approach assumes that the electricity markets are fully observable, which is not true under most situations, especially within deregulated electricity markets. POMDP is a methodology for decision making when the underlying process is modeled with HMM. POMDP naturally extends MDP by realizing that market states can only be partially observed by market-clearing prices and quantity. The following discusses the difference between MDP and POMDP, and the application of POMDP to the valuation and operation of generation assets.

MDP states of the underlying process are completely observable. A policy is a mapping from the set of observable states to the set of feasible actions. If both sets are assumed to be finite, the number of possible mappings is also finite. An optimal policy can be found by conducting a search over this finite set of mappings.

POMDP states, on the other hand, are only partially observable. Information from previous steps needs to be taken into consideration. All such information can be summarized by a probability distribution of the current state. In the literature, this probability distribution is often referred to as a belief state. The belief space is defined to be the set of all possible belief states. In a POMDP, a policy is a mapping from the belief space to the set of actions. The final solution of a POMDP is a mapping policy that defines the boundaries in belief state space where different decisions should be made. The belief state space is a continuous space, although the underlying process has only a finite number of states. This complicates the solution of a POMDP.

Improved computation efficiency could be gained by modifications aimed for specific applications, such as valuation of generation assets in electricity markets. Although the belief state space is continuous, the periodicity of electricity market mitigates the computation burden. If the assumption that an electricity market periodically returns to a known state holds, a POMDP could be constructed with known boundary conditions. With a

clearly defined boundary and limited time intervals involved, a POMDP can be easily solved. One example of those assumptions is that an electricity market returns to a known state with matched moderate demand and moderate online generation capacity at a given time interval, such as 2 AM. When applied to short-term operation decisions such as unit commitment and economic dispatch, physical constraints of generation assets also reduce the feasible actions space, thus reduces the computation burden of solving a POMDP.

The decision-making process shown in Figure 31 illustrates how POMDP is applied to valuation of generation assets for this work. POMDP models generation assets as a set of European options, which expire along the trading hours of electricity markets. At each time interval, the set of European options is valued and optimal decisions on the execution and expiration of options are made based on the belief state.

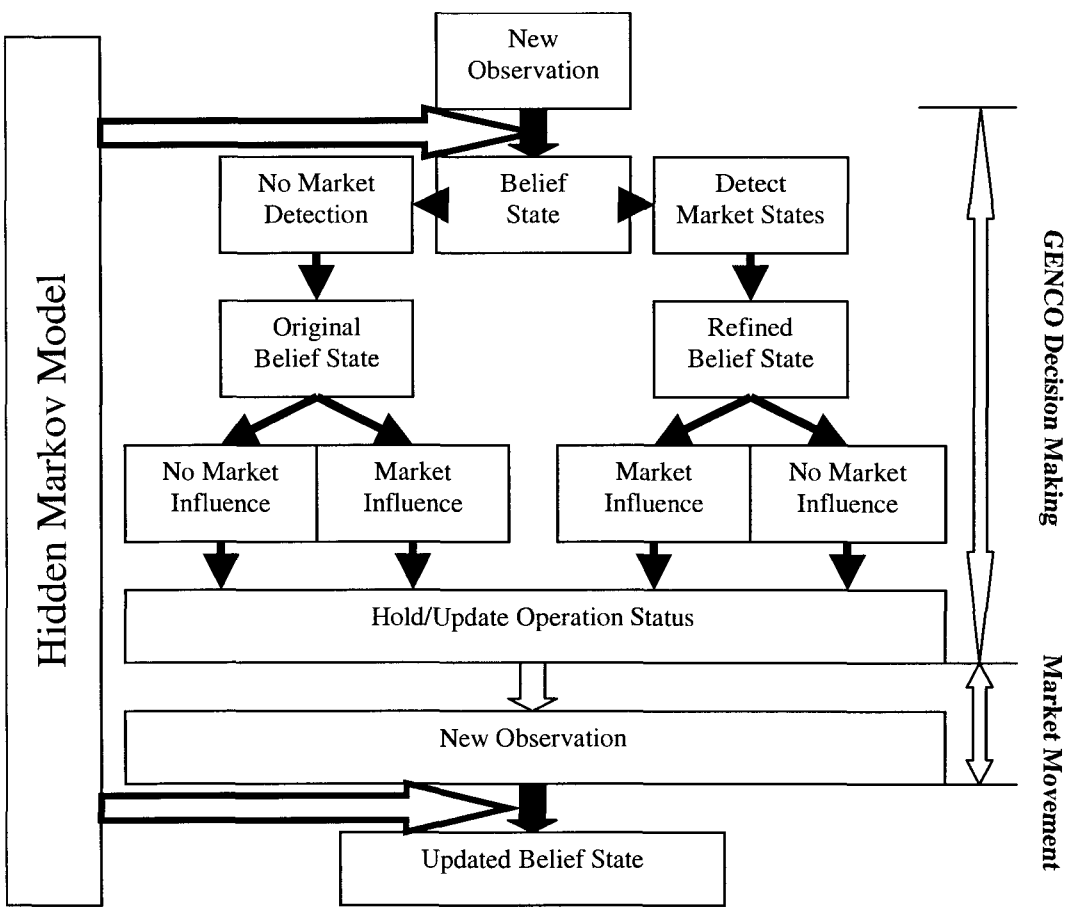


Figure 31: POMDP for Generation Assets Valuation

When a GENCO observes or receives new information, a new belief state is built based on a pre-structured HMM. The first decision is whether to execute call options to gather more information at certain costs. Such information helps to refine the belief state and facilitate the decision-making process. The following decision is whether to execute options to influence the market movement if possible. The last decision is whether to execute options to update generation units' operation statuses. Then the market takes over and new information is generated and observed. In fact, the call options on information gathering and market influencing could be in reverse sequence or parallel exclusive or parallel non-exclusive. Other options are also possible such as hedging/speculating on forward/future/option markets.

This section illustrates the application of MDP and POMDP with numerical examples. Table 17 defines one generator to be investigated. An hourly electric energy market is defined with an FHMM with underlying demand on electric power and supplied online generation capacity. Both the supply and demand are modeled to have three possible states including low, moderate, and high. Table 18 defines the states transition matrixes of demand on electricity and supplied online generation capacities. Depending on the states of demand and supplied online generation capacity, the prices on electric energy and ancillary services are assumed to follow normal distribution, the means, variances and daily pattern are also shown in Table 18. The prices of electricity, weather and insurance behave the same in the risk-neutral world and real world in such a way that the percentage changes in the underlying variable has no correlation with market return. Therefore, the actuarial approach for valuation is applicable to valuation of generation assets in that applying historical data to get the expected payoff and discount at the risk-free rate [7]. Binomial tree is employed to approximate the distribution of the price movement given a specific market state.

Table 17: Physical Characteristics of Generation Unit to be Valuated with POMDP

Generators Specifications	
Minimum Up Time, Ton (Hours)	1
Minimum Down Time, Toff (Hours)	1
Start-Up Time, Tup, (Hours)	1
Shut-Down Time, Tdown, (Hours)	1

Ramp Rate, Ramp, (MW/Hour)	100
Minimum Output Level, Qmin,(MW)	100
Maximum Output Level, Qmax,(MW)	200
Production Cost, $/MWh	32

Only one Markov chain is utilized for simplicity. More states and HMM chains could be used to increase the accuracy at cost of computational efficiency. Ancillary service market is also ignored for simplicity.

Table 18: Hidden Markov Model for Electricity Markets

Demand on Electric Energy States Transition Matrix			
	Low	Normal	High
Demand at T = Low	0.9229	0.0771	0
Demand at T = Normal	0.0199	0.9634	0.0168
Demand at T = High	0	0.1085	0.8915
Online Generation Capacity States Transition Matrix			
If Online Generation Capacity at Time T-1 = Low			
Demand at T = Low	0.9585	0.0315	0.0100
Demand at T = Normal	0.9434	0.0466	0.0100
Demand at T = High	0.0000	1.0000	0.0000
If Online Generation Capacity at Time T-1 = Normal			
Demand at T = Low	0.2097	0.7803	0.0100
Demand at T = Normal	0.1531	0.8369	0.0100
Demand at T = High	0.5898	0.4002	0.0100
If Online Generation Capacity at Time T-1 = High			
Demand at T = Low	0.0000	0.9900	0.0100
Demand at T = Normal	0.0000	0.9900	0.0100
Demand at T = High	0.0000	0.9900	0.0100
The states are numbered 1 to 9 and listed in parentheses.			
Price on Electric Energy (Normal Distributions, Mean, Normalized)			
Supply =	Low	Normal	High
Demand at T = Low	0.8794(1)	1. 0344(2)	1.3702(3)
Demand at T = Normal	1.0027(4)	0.9054(5)	0.9188(6)
Demand at T = High	1.1951(7)	1.2513(8)	1.3896(9)
Price on Electric Energy (Variance, Normalized)			
Supply =	Low	Normal	High
Demand at T = Low	0.0085(1)	0.0154(2)	0.0025(3)
Demand at T = Normal	0.0086(4)	0.0204(5)	0.0025(6)
Demand at T = High	0.0170(7)	0.0216(6)	0.0069(9)

Daily Price Pattern								
Time (Hour)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Price ($/MWh)	15	15	15	15	17	19	21	23
Time (Hour)	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Price ($/MWh)	25	27	30	34	40	50	40	34
Time (Hour)	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Price ($/MWh)	30	27	25	23	21	19	17	15

The electricity market states are categorized into four groups as listed in Table 19. Each group has a price interval, which is based on the expected price for the states in that group. Table 19 also gives the steady-state probability for each state and group using the

Chapmen-Kolomogorov Equation. This example is simplified by assuming that a group of states, but not the individual state, can be identified from the observed market prices. It is also possible to use belief state defined as a distribution about individual states with Maximum-Likelihood algorithms.

Table 19: Categorization of Market States and Corresponding Long Term Probabilities

P(Group_1) = 0.4166			P(Group_2) = 0.5368	
P(S_1)	P(S_5)	P(S_6)	P(S_2)	P(S_4)
0.1464	0.204	0.0662	0.0355	0.5013
P(Group_3) = 0.0504			P(Group_4) = 0.0025	
P(S_7)	P(S_8)		P(S_3)	P(S_9)
0.0443	0.0071		0.0018	0.0007

This example focuses on the operation of the generation unit. The generation unit is assumed to be ON and producing at 300MW/hour at 15PM, and the electricity market state is estimated to be group_1. The generation unit needs to be turned down since the highest price at 20PM and after is lower than the production cost of the unit. The Markov price lattice is shown in Figure 32.

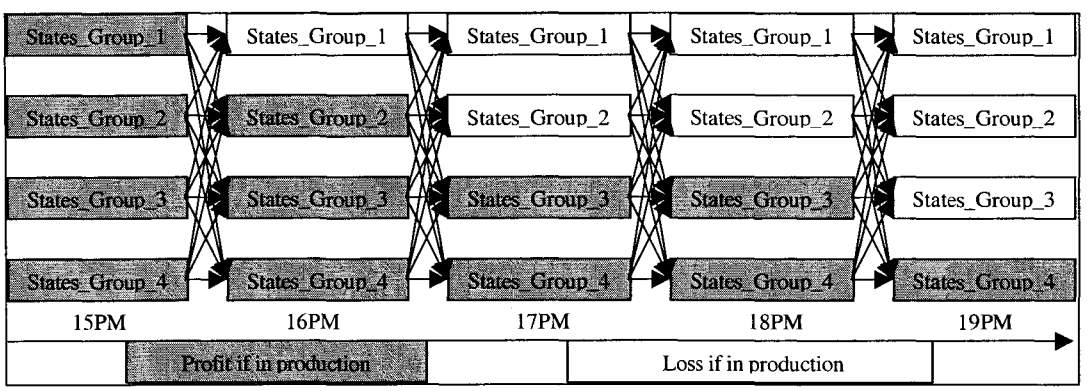


Figure 32: Markov Prices Lattice for POMDP

Table 20 gives the optimal operation policy solved by POMDP. Table 21 gives the optimal operation policy if perfect information is available, which is also the solution to the corresponding MDP problem. It is noticed that the optimal operation policy for a state group is not always consistent with the optimal operation policies for individual states in such a

group. Under such situations, more information about the current market state provides more insight on the decisions to be made.

Table 20: Optimal Operating Policy Based on POMDP

Time	15PM	16PM	17PM	18PM	19PM
Group_1	On->Off	Off	Off	Off	Off
Group_2	On->On	On->Off	Off	Off	Off
Group_3	On->On	On->On	On->On	On->Off	Off
Group_4	On->On	On->On	On->Off	On->Off	On->Off
State A->State B: If State A then State B					

Table 21: Optimal Operation Policy Based on MDP (Perfect Information)

Time	15PM	16PM	17PM	18PM	19PM
State_1	On->Off	Off	Off	Off	Off
State_5	On->On	On->Off	Off	Off	Off
State_6	On->On	On->Off	On->Off	On->Off	On->Off
State_4	On->Off	Off	Off	Off	Off
State_2	On->On	On->Off	Off	Off	Off
State_7	On->On	On->On	Off	Off	Off
State_8	On->On	On->Off	On->Off	On->Off	Off
State_3	On->On	On->Off	On->Off	On->Off	Off
State_9	On->On	On->On	On->Off	On->Off	On->Off

4.2.3 Value of Information

It is shown that the optimal operation policy under MDP (Perfect Information and Observability) is different with the optimal operation policy under POMDP (Imperfect Information and Partial Observability). For an MDP, a decision maker is a "state taker," who observes and forecasts the state transition and makes decisions based on its perception. For a POMDP, a decision maker makes decisions based on incomplete information. The decision-maker will be better off if more information about the system can be gained at reasonable cost. A decision maker will also be better off if the transition of system can, to some extend, be controlled. Comparing the results given by MDP, POMDP could identify the value of information or the system manipulating.

This section focuses on the value of information, both perfect information and imperfect information. Table 22 and Table 23 provide the expected value for one MW generation capacity of the investigated generation unit in each state group or individual state

for a specific hour. It is shown that more information on the market improves the profitability of generation assets. The value of such information depends on the physical capabilities of the investigated generator and the electricity markets. If the market is in state 5, where both supply and demand are moderate, the optimal operation is to keep the generation unit on. If the GENCO only knows that market is in state group_1, the optimal operation is to shut generation units off. Keeping generation unit on provides extra future. In this case, turning generation unit off provides only current profit, $9.3760/MWh. Keeping the generation unit on provides $12.0543/MWh, a difference of 28.56%.

Table 22: Generation Unit Value Based on POMDP ($/MWh)

Time	15PM	16PM	17PM	18PM	19PM
Group_1	4.048	0	0	0	0
Group_2	10.8760	2.6307	0	0	0
Group_3	29.1599	13.7307	5.10234	1.0264	0
Group_4	30.4773	16.1334	9.3970	5.2573	2.4875

Table 23: Generation Unit Value Based on MDP, Perfect Information ($/MWh)

Time	15PM	16PM	17PM	18PM	19PM
State_1	3.1760	0	0	0	0
State_5	12.0543	3.1696	0	0	0
State_6	26.1616	14.5868	9.1060	4.9954	2.2550
State_4	4.2160	0	0	0	0
State_2	9.4875	2.0918	0	0	0
State_7	12.7926	2.5691	0	0	0
State_8	20.4768	8.6334	3.8530	0.2677	0
State_3	20.3137	10.5442	5.5390	1.7851	0
State_9	31.7262	18.6393	9.6880	5.5192	2.7400

Equation 17 illustrates how to find the expected value of perfect information for a single market state or a market state group. Imperfect information could only define a probability distribution over a set of states, which provides less value. The value of imperfect information could be obtained as defined in Equation 17. It is also observed that more information may not provide any value when the optimal operation policy for a state group is consistent with the optimal operation policies for all individual states. The optimal strategy for call option on more information should be left expired under such situations. The absence of impact of physical constrains of generation assets in this example is due to the fact that generation unit is a very fast unit.

$$
\begin{aligned}
&\textit{Value of Perfect Information for a Market State}\\
&= \textit{Value by MDP for an individual state}\\
&\quad - \textit{Value by POMDP for an individual state}\\
&\quad V_{perfect,state\_1} = 3.176 - 3.176 = 0\\
&\quad V_{perfect,state\_5} = 12.0543 - 9.3760 = 2.6783\\
&\quad V_{perfect,state\_5} = 26.1616 - 22.808 = 3.3536\\
&\textit{Expected Value of Perfect Information for a State Group}\\
&= \sum_{i=1}^{States\ in\ a\ Group} Probabilty\_State\_In\_Group *\\
&\qquad \textit{Value of perfect information for a market state}\\
&\quad EV_{perfect,state\_group\_1} = 1.8445 =\\
&\quad 0.3514*0 + 0.4897*2.6784 + 0.1589*3.3536
\end{aligned}
$$

Equation 17: Value of Information Based on MDP and POMDP

4.3 Investment in Generation Assets

Short-term valuation of generation assets assumes HMM with fixed structure and deterministic parameters, while long-term valuation of generation assets confronts time-varying Markov process structure and parameters. The structure of a Markov process refers to how many states an electricity market could enter, the relationship of all the states and so on. The parameters of a Markov process refer to the transition probability matrix and the price distributions given a specific system state. This section proposes to capture the uncertainties of model parameters with fuzzy sets and fuzzy numbers. The variability of model structure is proposed to be treated using scenarios analysis.

The long-term valuation of generation assets, namely the valuation of investment in generation assets, requires a more fundamental model for electricity markets. This section reproduces example in Chapter 3, and applies Markov Model to Location Marginal Pricing (LMP) electricity markets. The HMM is modified to capture the long-term fuzziness.

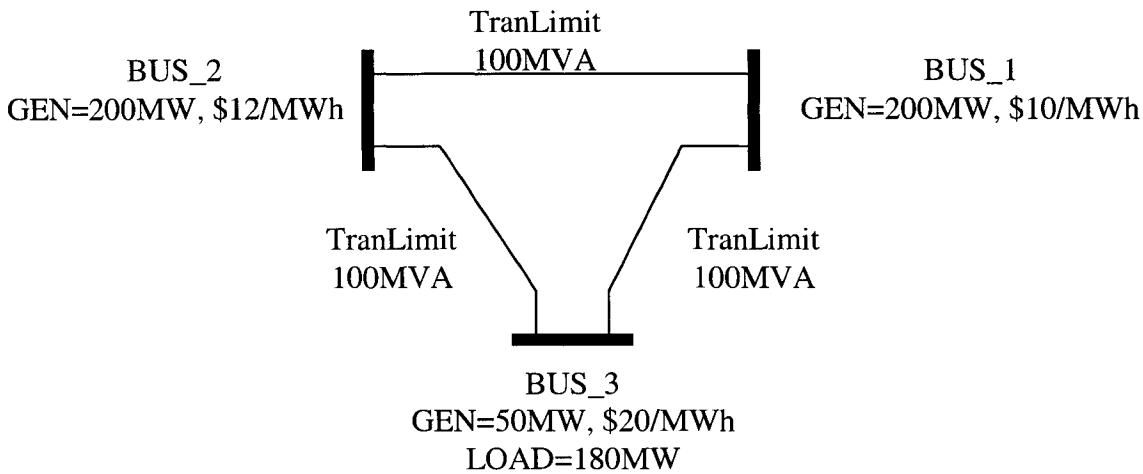


Figure 33: Simplified Generation Assets Investment Transmission Network

The to be valuated generation assets are assumed to located at Bus_3 shown in Figure 33. This example analyzes one scenario, which implicitly assumes that the to be built generation plant at Bus_3 is a price-taker and will not affect the LMP at Bus_3. This assumption is only true provided two underlying assumptions hold:

The "to be built" generation plant are more efficient than generators at Bus_1 and Bus_2, and has a corresponding production cost lower than $10/MWh

The 180MW demand shown in Bus_3 is residual demand after the to be built generation plant is committed.

The price-taker assumption is employed to simplify the discussion. If the to-be-built generator could have impacts on the LMP, the system state and state transition matrix should be extended to include the impact of new generators. This work assumes a price-taker 500MW base generator producing at $9/MWh while the total demand at Bus_3 ranges from 550MW to 680MW to be consistent with the example. The lower production cost of the to-be-built base generator leads to an always in-the-money call option. The optimal policy would be keeping the unit on whenever possible. This assumption also justifies the ignorance of most generator operating time constraints. For a base generation unit investigated in this scenario, most of the profit is gained during peak-hours. This example focuses on the impact of transmission network.

The Markov chain for valuation is modeled to include four market states jointly determined by the transmission network states and the investigated generation unit's operating status. The transmission network states space includes two states, depending whether the transmission line from Bus_1 to Bus_3 is congested or not. Each state is defined by a two-element vector as shown in Equation-11.

$$State\_1(Non-congested):\begin{bmatrix}1\\0\end{bmatrix} State\_2(Congestion):\begin{bmatrix}-1\\2\end{bmatrix}$$

Equation 18: Contribution Factor State Space for Generation Asset Investment Example

If there are more than two adjacent buses that have impacts on the LMP of the investigated bus, a multiple-element vector should be defined. The state space of the generation unit is assumed to include only two states: On and Off. This simplification is consistent with the base generator assumption. A base generator normally operates either at full capacity or off. If detailed physical constraints are to be included in the formulation, the state space of the generation unit needs to be expanded to include different output levels and transition states.

The market state transition is jointly determined by the demand level at Bus_3, generation offers from GENCOs at Bus_1 and Bus_2, and the transmission capacity of the given regional transmission network. Assume the estimated crisp transition matrix as shown in Equation 19.

$$Crisp\_State\_Transition\_Matrix$$

$$\begin{bmatrix} & State\_1\_On & State\_2\_On & State\_1\_Off & State\_2\_Off \\ State\_1\_On & 0.8 & 0.2 & 0 & 0 \\ State\_2\_On & 0.2 & 0.8 & 0 & 0 \\ State\_1\_Off & 0 & 0 & 0.8 & 0.2 \\ State\_2\_Off & 0 & 0 & 0.2 & 0.8 \end{bmatrix}$$

Equation 19: Crisp State Transition Probability Matrix for Generation Asset Investment

A probability price distribution could capture the randomness of LMP prices for a given system state. Another alternative to capture the fuzziness of prices for a given system state is to utilize fuzzy numbers.

Fuzzy numbers could enter the modeling of LMP by many ways. This work approaches modeling of LMP by construing a fuzzy states transition matrix to capture the fuzziness of electricity market movements. Fuzzy numbers are employed to define the possibilities of electricity market transition. The incorporation of fuzzy numbers leads to the formulation of generation assets valuating problems as fuzzy real call options. A fuzzy real option could be formulated as a fuzzy MDP, which could be solved with fuzzy linear programming.

There are two approaches to implement fuzzy linear programming:

- Zimmermann's approach [55]
- Tanaka and Asais (T&A's) approach [56]

Both Zimmermann's and T&A's approaches can be used to solve fuzzy linear programming problems, where each of them is targeted at different problem structures. More discussion can be found [57]. Once the fuzzy linear programming problem is solved, then the following results are expected [58][59]:

- Optimal operation strategies such as unit commitment and economic dispatch
- Sensitivity analysis of physical constraints including transmission and generation parameters
- Dynamic hedging and portfolio management insights

This work adopts T&A's for it allows sensitivity analysis on the constraints, which could not be achieved neither by regular sensitivity analysis nor parametric analysis. This feature is of great value to long-term scenarios analysis. T&A's approach uses fuzzy numbers to model all the coefficients in a LP problem—namely coefficients in constraint matrix, objective function, and right-hand side vector as shown in Equation 20.

$$\begin{aligned} \min Z = C'X \quad &\Rightarrow \begin{pmatrix} \tilde{\mathbf{c}}^T & -\tilde{z} \\ -\tilde{\mathbf{A}} & \tilde{\mathbf{b}} \end{pmatrix} \mathbf{x} \geq 0 \\ s.t. AX \leq b \quad &\Rightarrow Fuzzy\ Functions \\ X \geq 0 \quad &\Rightarrow X \geq 0 \end{aligned}$$

Equation 20: T&A Approach for Fuzzy Linear Programming Formulation

Fuzzy numbers are special cases of fuzzy sets and this work applies triangle fuzzy numbers as shown in Figure 34. A symmetric triangle fuzzy numbers is defined by its center α and half-width c.

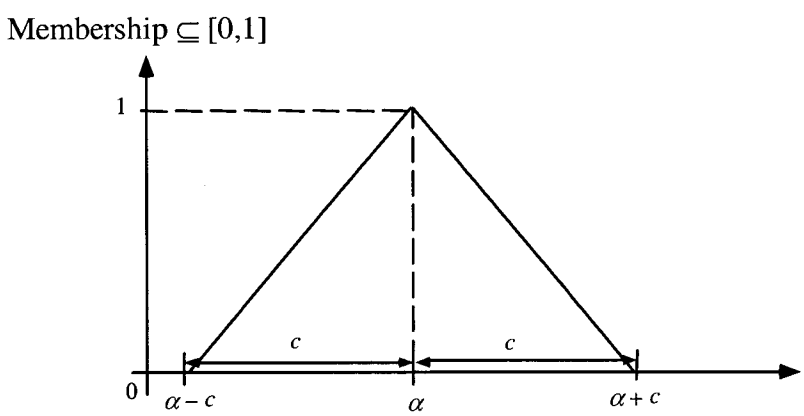


Figure 34: Symmetric Triangle Fuzzy Number

A fuzzy function is a generalization of a classical crisp function and it can be defined in many ways. This work adopts a classical definition for fuzzy function of fuzzy numbers as illustrated below. Assume a function $Y = f(X, A)$ with its inputs, outputs and parameters $X, Y, and\ A$ all defined as crisp numbers. Given a parameter set A, this function maps X to Y. Given a fuzzy number $\tilde{A}$, a fuzzy function $f: X \rightarrow \varphi(Y)$ mapping X to $\varphi(Y): set\ of\ all\ fuzzy\ subsets\ on\ Y$ is defined by a membership function on Y.

$$\tilde{Y} = f(x, \tilde{A}),$$

Membership function for $\tilde{Y}$

$$\mu_Y(y) = \begin{cases} \max\limits_{\{a \mid y = f(x,a)\}} [\mu_A(a)], & \{a \mid y = f(x,a)\} \neq \varnothing \\ 0, & otherwise \end{cases}$$

Equation 21: Definition of Fuzzy Function

This work begins with deterministic price presentations, and then extends to fuzzy numbers. The cost/payoff for different combinations of system states and decisions are shown in Equation 22. The non-feasible decisions are assigned a large cost M to preserve infeasibility in the solution. The startup cost is assumed to be 1000 while the shutdown cost is assumed to be zero.

Cost / Payoff

$$\begin{bmatrix} & Stay\_Off & Stay\_On & Turn\_On & Turn\_Off \\ State\_1\_On & -M & 500 & -M & 0 \\ State\_2\_On & -M & 2500 & -M & 0 \\ State\_1\_Off & 0 & -M & -1000 & -M \\ State\_2\_Off & 0 & -M & -1000 & -M \end{bmatrix}$$

Equation 22: Crisp Payoff Matrix for Generation Asset Investment Example

The valuation of the to-be-built generator is formulated as a MDP problem and solved as a Linear Programming (LP) problem. Equation 23 defines variables for the problem while the crisp LP problem is defined in Equation 24.

$$
\begin{aligned}
&System\ State\ i = 1,2,3,4\ for\ Non\text{-}Congested, Congestion, \\
&\qquad Unit\_On, Unit\_Off \\
&Decision\ k = 0(StayOff), 1(StayOn), 2(TurnOff), 3(TurnOn) \\
&Y_{i,k} = Prob(State = i\ and\ Decision = k) \\
&\qquad Steady\ State\ Unconditional\ Probability \\
&C_{i,k} = Cost\,/\,Payoff\,(State = i\ and\ Decision = k) \\
&P_{i,j} = Prob(State = i\ and\ Transition\_To = j) \\
&\qquad System\ State\ Transition\ Probability \\
&D_{i,k} = Prob(Decision = k \mid State = i\ ) \\
&\qquad Randomized\ Policy,\ Conditional\ Probability
\end{aligned}
$$

Equation 23: Variables Definition of Crisp Linear Programming Problem for Generation Assets Investment

$$
\begin{aligned}
&Maximize\ Z = \sum_{i=1}^{2}\sum_{k=0}^{1} C_{i,k} Y_{i,k} \\
&S.T. \qquad \sum_{i=1}^{2}\sum_{k=0}^{1} Y_{i,k} = 1 \\
&\qquad \sum_{k=0}^{1} Y_{j,k} - \sum_{i=1}^{2}\sum_{k=0}^{1} Y_{i,k} P_{i,j}(k) = 0\ \forall j = 0..M \\
&\qquad Y_{i,k} \geq 0 \\
&Policy \qquad D_{i,k} = \frac{Y_{i,k}}{\sum_{k=0}^{1} Y_{i,k}}
\end{aligned}
$$

Equation 24: Crisp Linear Programming Problem for Generation Assets Investment

This work then incorporates fuzzy information to capture the uncertainties and fuzziness of long-term forecast. The movements of prices of electricity are represented using fuzzy numbers, and those fuzzy numbers contribute to a fuzzy objective. The constraints defining the market movements are represented with fuzzy functions, where fuzzy transition matrix is employed. The fuzzy state transition matrix is defined by replacing probabilities with possibilities using fuzzy numbers as in Equation 25.

$$Fuzzy\_Transition\_Matrix$$

$$\begin{bmatrix} & State\_1\_On & State\_2\_On & State\_1\_Off & State\_2\_Off \\ State\_1\_On & 0.\tilde{8} & 0.\tilde{2} & 0 & 0 \\ State\_2\_On & 0.\tilde{2} & 0.\tilde{8} & 0 & 0 \\ State\_1\_Off & 0 & 0 & 0.\tilde{8} & 0.\tilde{2} \\ State\_2\_Off & 0 & 0 & 0.\tilde{2} & 0.\tilde{8} \end{bmatrix}$$

Equation 25: Fuzzy State Transition Possibility Matrix

The costs/payoffs are defined for a combination of system state and feasible decision depend on the fuel prices, GENCO's O&M cost, and other economic drivers. This work captures the fuzziness of costs/payoffs by fuzzy numbers as shown in Equation 26.

$$Cost\,/\,Payoff$$

$$\begin{bmatrix} & Stay\_Off & Stay\_On & Turn\_On & Turn\_Off \\ State\_1\_On & -M & \tilde{500} & -M & 0 \\ State\_2\_On & -M & \tilde{2500} & -M & 0 \\ State\_1\_Off & 0 & -M & -\tilde{1000} & -M \\ State\_2\_Off & 0 & -M & -\tilde{1000} & -M \end{bmatrix}$$

Equation 26: Fuzzy Payoffs of Generation Assets Investment

The fuzzy LP problem is as shown in Equation 27

$$\begin{aligned} & Maximize\ Z = \sum_{i=1}^{2}\sum_{k=0}^{1} \tilde{C}_{i,k} Y_{i,k} \\ & S.T. \qquad \sum_{i=1}^{2}\sum_{k=0}^{1} Y_{i,k} = 1 \\ & \qquad \sum_{k=0}^{1} Y_{j,k} - \sum_{i=1}^{2}\sum_{k=0}^{1} Y_{i,k}\tilde{P}_{i,j}(k) = 0 \ \forall j = 0..M \\ & \qquad Y_{i,k} \geq 0 \\ & Policy \qquad D_{i,k} = \frac{Y_{i,k}}{\sum_{k=0}^{1} Y_{i,k}} \end{aligned}$$

Equation 27: Fuzzy Linear Programming Problem for Generation Assets Investment

To be noticed that coefficients in constraint matrix $\tilde{P}_{i,k}$ and objective function $\tilde{C}_{i,k}$ are defined by fuzzy numbers. The fuzzy linear programming problem shown in Equation 27 could be transformed into a crisp non-linear problem as shown in Equation 28. The result of this transformation is a non-linear programming problem, which can be solved by the successive solving of linear programming problems.

$$\begin{aligned}
&Maximize\ \lambda \\
&s.t.\ \ (\alpha_i - \lambda c_i)x \geq 0, \\
&\alpha_i, c_i\ are\ the\ center, width\ of\ tri-angle\ fuzzy\ number
\end{aligned}$$

Equation 28: Transformation of Fuzzy LP into Non-Linear Programming Problem

The solution for the crisp LP problem shown in Equation 24 and fuzzy LP problem shown in Equation 27 is given in Table 24.

Table 24: Solutions for Crisp and Fuzzy Linear Programming Problems (Generation Assets Investment)

	Crisp LP Problem	Fuzzy LP Problem
Operation Policy	Always On	Always On
Generation Value	1500/MWh	*Triangle Fuzzy Number* $(1500, 300)/MWh$

The results shown in Table 24 are for one scenario within one single time interval as shown in Figure 35. The same approach could apply to multiple scenarios and time intervals, as well as intermediate and peak generation assets.

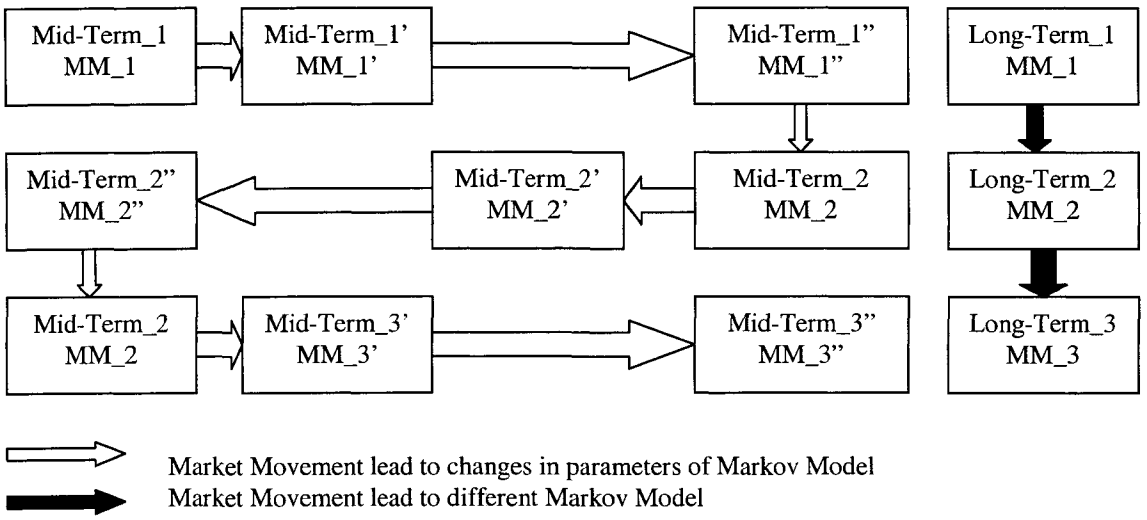


Figure 35: Long Term Markov Model for Electricity Markets

The fuzzy-number based real option analysis provides potential investor an efficient valuation strategy for generation assets. Multiple Markov models allow incorporating significant market movements, which is common for the long time span of generation assets. The decomposition of LMP movements into physical and economic drivers provides a framework to capture the location-depend and time-varying value of generation assets. The location-depend and time-varying value of generation assets determines the site and technology of to be invested generation assets. The fuzzy approach enhances the modeler's capability to incorporate subjective judgments when long-term forecast is not available or could only be obtained at prohibitive cost. The fuzzy feature also provides an efficient tool for comparing different investment opportunities in generation assets.

Chapter 5: Conclusion

Generation assets valuation and investment are of critical importance for electric power industry re-regulation. Real option analysis provides a promising methodology to capture and quantify the embedded operating flexibilities of generation assets. Improvements and refinements on applying real option analysis to valuating generation assets have been extensively investigated. Most of the efforts are two-folded, enforcing the physical constraints of operating generation assets and capturing the unique features of electricity prices movements.

Electricity, as a special commodity, demonstrates unique features on its markets movements. The time-varying and location-dependent value of electricity should be captured by an open framework, which combines both physical and market information efficiently.

Generation assets, as real assets, embed structured operating flexibilities and constraints. The short-term operating constraints, combined with volatile and incomplete information, requires a valuation tool providing tractability and insights.

This work proposes an integrated valuation framework, which is built to address not only the constraints of operating generation assets, but also their impacts on the market movements. The Hidden Markov Model combines the strengths of both Mark-To-Market approach and production cost modeling approach. Information coming both from the market and other sources are integrated into an open framework. This framework is rich in its structure to allow refinements if warranted by valuation purposes and availability of data on markets. Electricity markets could be decomposed into electric energy and ancillary services markets in the HMM framework. This HMM framework also allows to decompose the markets drivers in different ways, supply v.s. demand forces, and physical v.s. economic forces. The HMM framework is also extensible from short-term to long term. The incorporation of fuzzy sets and numbers allows the incorporation of subjective judgments and capture of market uncertainties and fuzziness

This work proposes real option analysis to valuate generation assets. Generation assets valuation problems are formulated as Partially Observable Markov Decision Problem (POMDP), which is identified as a real option analysis approach. The incorporation of incomplete information identifies the inconsistence between electricity markets states and observed prices. It is shown that optimal operation of generation assets could be achieved based on the electricity markets states instead of observed markets prices. Long-term generation assets investment is formulated as fuzzy MDP. The incorporation of fuzzy information facilitates incorporating subjective forecast.

This work incorporates more information; both market information and engineering insights. It takes both the operating flexibilities and operating constraints into consideration when valuating generation assets. This approach could be used to establish future markets for electricity markets.

Appendix

A.1 Regulatory History of the U.S. Electric Power Industry

The U.S. electric power industry had operated un-regulated, regulated and it is now being re-regulated. A review of such an evolution sheds light on the on-going re-regulation worldwide. Before re-regulation, electric power generation, transmission, and distribution had been considered to be a "natural monopoly." It was believed that the electric power industry operates most efficiently as a monopoly because of its decreasing average long-run costs due to economy of scale. Today, there is a widespread view among legislators, regulators, industry analysts, and economists that the generation segment of electric power industry in today's environment would be more efficient and economical in a competitive market. In contrast, transmission and distribution segments will remain regulated and noncompetitive while generation segment of the electric power industry is being restructured. The electric power industry is currently in the midst of a transition from a vertically integrated and regulated monopoly to horizontally integrated entities in a competitive market where both customers and suppliers trade freely. Although the electric industry keeps evolving, the economic drive behind all those re-structuring remains the same, seeking cheap and reliable electric power supply.

The electric power industry began with Edison Electric Illuminating Company and operated un-regulated from 1882. Early electric power companies were inefficient and redundant in the services they provided. Both technical and economic issues contributed to the inefficiency. Companies used different equipments, voltages, and frequencies so their systems were not compatible, and were isolated. All companies operated on small scales, and provided all services by themselves, and no economy of scale was realized. One single company generated, and delivered the electric power supply to customers by its own point-to-point distribution lines. In order to operate, the companies had to acquire franchise rights from the local municipalities. Electric power companies frequently operated under

nonexclusive franchises that were often in competition with one another. The award of franchises was often based on bribing government officers and no transparency and equality was presented. Franchise territories differed greatly, from a single block to the entire city. The franchise process kept the industry fragmented and inefficient. The issuance of numerous short-term franchises created an uncertain, chaotic operating environment. It was during these early years that entrepreneurs like Samuel Insull, who had worked with Edison, gave his attention to utility operations and began to shape and define important economic concepts which still govern modern utility planning and pricing.

Early industry leaders recognized that electric power companies suffered from high fixed capital costs as a result of the heavy investment needed to finance central generating plants and distribution systems. At the same time, the variable costs were relatively low. Fixed costs reflected the fix amount of investment that must be paid regardless of output, and they included electric power system construction and equipment. Variable costs were those that varied with the level of electricity output and included fuel expenses. Samuel Insull understood that with more customers on an electric power system, more revenue was generated spreading out fixed costs. With the development of the electricity demand meter, Samuel Insull reduced prices of electricity and aggressively marketed to attract more customers. The electricity demand meter measured not only a customer's peak electricity demand, or the "share" of fixed cost required for usage, but also the actual electricity used. Samuel Insull set the price of electricity to cover both of these costs: fixed (demand share) and variable (electric energy). Ancillary services were bundled together with electric energy and no separate pricing was implemented. Later, this pricing practice was replaced by a flat rate in the regulated electric power industry. Other pricing practices had been proposed, such as differentiating prices between peak hours and non-peak hours to provide incentives for demand side management. Re-regulation may introduce new pricing structure, which are not necessarily to be one of the above. An Electricity Service Company (ESCO) can devise new price structures to serve customer utilizing the load management programs effectively.

Early analyses showed that the more time a generation plant was in use, the higher the profit with a lower average cost to customers. Since electricity could not be stored the trick was to find the right mix of customers to utilize the plant for as much of a day as possible.

For example, in order to maximize plants to their fullest, one might supply the early morning and late afternoon streetcar load, an evening residential lighting load, and a business and industrial load between the two streetcar-peak loads. Samuel Insull realized that the same three loads could be served by one plant instead of three different plants that were then being used. In addition to load diversity, it was discovered that technically increased efficiency could be realized through economy of scale. One large centrally located power station could be operated more cheaply than numerous isolated small generating units. However, both load diversity and economy of scale required a well-connected electric power delivery system. The formation of the electric power industry was heavily influenced by the inherent advantage in serving many customers, the desirability of load diversity, and building to achieve economies of scale. Today, with the advent of Combine Cycle Gas Turbine (CCGT) and some renewable energy plants, smaller generation units can again be cost effective and more environment friendly. Also, new technology makes the building of a power plant quicker and cheaper. Such technology advances make the competition with the electricity generation segment possible and worthwhile. This technical trend has been the one of the motivations for re-regulation and will influence the re-construction of the electric power industry.

Before regulation, electric power companies frequently found that it was difficult to maintain investor confidence and attract adequate capital. This was attributable to both the dubious franchise process, which made operation of the utility over the long term an uncertain prospect, and the low returns investors received. Early industry leaders began to think that if the franchise granting process and the rates charged by electric power companies were overseen by a nonpartisan state agency instead of a city council, financing might be easier and cheaper to obtain. A higher level agency not only led to a relatively stable franchise, which mitigated the long-term operation risk and lowered the cost of capital, but also made the integration of isolated electric power systems feasible. Such a belief led to the regulation by state agencies after 1898. With re-regulation, operational risks present themselves again and bring market risk to the investment in utilities. The operation risks of the electric power industry require a mechanism to allocate, share and management of such

risks. Also, Federal Energy Regulation Committee (FERC) is pursuing more regulation authority to meet the challenge of operating a gigantic electric power system.

In 1898, Samuel Insull proposed that electric power companies be regulated by state agencies that would establish rates and set service standards. The idea became increasingly appealing to Investor-Owned Utilities (IOU) in the face of public enthusiasm for the growth of municipal electric systems. Privately owned electric power companies surmised that the public might be more supportive if their companies were regulated so that customer interest would be protected. By 1916, 33 states had regulatory agencies in U.S. Early regulation of the industry proved beneficial to both the electric power companies and to their customers who got reliable, reasonably priced service without the uncertainties caused by duplicate services and inefficient operations. It is believed by many that re-regulation will benefit both the electric power industry and its customers because many regulation-induced distortions will be removed and the re-regulated electric power industry is motivated to lower the cost of meeting demand.

As the regulation of the electric industry evolved, an electric utility began to service as a natural monopoly because a single company providing electricity was more economically efficient for its eliminating of duplication of service, equipments and economy of scale. Regulated electric utilities assumed certain common characteristics of their rights and obligations. These included:

- Assignment of Franchise or Service Territory
- An exclusive franchise grants an electric utility to serve customers within a designated geographic area, known as its service territory, often for a relatively long time;
- Obligation to Serve

In return for the exclusive franchise, an electric utility is required to serve all existing and future customers equally and at a reasonable rate, which is under the authority of regulators;

By 1935, utilities had seen dramatically growth under the regulation of states governments. The electric power industry changed dramatically both technically and economically. Bigger and more capital-intensive electricity generation units were built to

pursue economy of scale. High voltage long distance transmission networks began to emerge, which connected existing intrastate electric power networks into interstate electric power networks. Electric utilities operating in different states were connected together, and coordinated their operation. Often those connected utilities were owned and controlled by holding companies, companies with controlling shares in other companies producing services and goods. Holding companies performed the coordination of interstate transmission of electricity internally, and provided common services to its holding utilities such as financing, maintenances of equipments. Three huge holding companies, along with more than 100 other holding companies, existed and produced half of the electricity in the U.S. The lack of federal regulation agents and the holding companies' size and complexity made industry regulation and oversight control by the states agents impossible. Federal Trade Commission (FTC) criticized that many holding companies were raising the cost of electricity to consumers. The Securities and Exchange Commission (SEC) also investigated this matter and publicly charged that the holding companies had been guilty of stock watering and capital inflation, manipulation of subsidiaries, and improper accounting practices. After re-regulation, the financing practice of the electric power industry comes again into focus, especially after Enron's accounting scandal.

Public Utility Holding Company Act (PUHCA) of 1935 aimed at breaking up the unconstrained and excessively large holding companies that then controlled the U.S. electricity and gas distribution networks. Under PUHCA, the SEC was charged with the administration of the Act and the regulation of the holding companies. One of the most important features of the Act was that the SEC was given the power to break up the massive interstate holding companies by requiring them to divest their holdings until each became a single consolidated system serving a circumscribed geographic area. Technically, the connected electric power systems were divided into control areas based on the strength of their electrical connection. Each control area included a set of roughly balanced generation units and loads with relatively strong connections, while control areas connected with each other with relatively weak connections. One control area coordinated the production, transportation, and consumption of electricity within its own reaches, and interchanges among control areas. One utility might have one more than one control areas, and the

interchanges of electricity between different utilities were implemented by bilateral contracts. Another feature of the PUHCA 1935 permitted holding companies to engage only in business that was essential and appropriate for the operation of a single vertically integrated utility. This latter restriction practically eliminated the participation of non-utility companies in wholesale electric power sales. The law contained a provision that all holding companies had to register with the SEC, which was authorized to supervise and regulate the holding company system. Through the registration process, the SEC decided whether the holding company would need to be regulated under or exempt from the requirements of PUHCA. The SEC also was charged with regulating the issuance and acquisition of securities by holding companies. Strict limitations on intra-system transactions and political activities were also imposed. PUHCA put utilities under more regulation and kept the cost of supplying reliable electric power relatively low. During today's worldwide re-regulation, some countries are alert to the holding company issues, namely the high concentration of generation assets. For example, Australia requires the divested generation capacities to be sold to different entities. In the U.S, no such requirements are in operation now. However, market power and market manipulation are under intensive monitoring.

During 1970s, the U.S. electric power industry was hit by severe unforeseen changes. Technology failed to make bigger nuclear generation units more efficient and the claim that electric power would be too cheap to measure was proved to be wrong. Another change was that the pattern of load increase changed from exponential increase pattern to saturated increase pattern. Those two changes resulted in less-utilized capital-intensive electricity generation units. The following oil embargo of 1973 led to the high cost of input for electric utilities. The poor planning of utilities further deteriorated the unfavorable impacts, and increased the cost of electric power. Under the cost-based regulation, utilities managed to convince the regulators to increase the rate of service and recouped cost from customers. All those changes led to Public Utility Regulation Policies Act (PURPA). PURPA stipulated that electric utilities had to interconnect with and buy, at the utilities' avoided cost, capacity and energy offered by any non-utility facility meeting certain criteria established by FERC. PURPA allowed some non-utilities to enter the franchised electric and gas utilities. The concept of natural monopoly was removed from mind set. PURPA also started the

disintegration of electric utilities from vertically orientated to horizontally integrate by contracts. PURPA was designed to encourage the efficient use of fossil fuels in electric power production through co-generators and the use of renewable resources through small power producers. The main objective was reducing the US's dependence on foreign oil but not re-regulation. Although PURPA contributes to the deployment of renewable energy under regulation, it hinders the incorporating of renewable energy into electricity markets when re-regulation began. For example, wind generators are still qualified as Small Power Producer Qualifying Facility (SPP QF) under PURPA. Thus wind generators hesitate to enter the uncertain electricity markets. California Independent System Operator (CAISO) proposed Participating Intermittent Resource Program (PIRP) to encourage wind generators to join the CA electricity market.

New technologies and successful re-regulation of several 'regulated natural monopolies' such as natural gas, airlines, and telecommunications motivated the re-regulation of the electric power industry. Energy Policy Act (EPACT) of 1992 opened access to transmission networks and exempted certain non-utilities from the restrictions of the Public Utility Holding Company Act (PUHCA) of 1935. In 1996, FERC issued Order 888 that further opened transmission access to non-utilities, thereby establishing wholesale competition. FERC Order 889 requires electric utilities to establish electronic systems to share information about available transmission capacity. A few electricity markets were set up and put into operation, such as CAISO, PJM, ERCOT, BETA, European Internal markets, Australian Victoria electricity markets among others. While all those electricity markets are trying to promote competition, they pursue it in different ways by adopting different market frameworks such as electric power pool, exchange, bi-lateral contracts and others. The searching for better framework for re-regulation never stops. The U.S. electric power industry has been searching for an optimal re-regulation framework. The creation of ISO (Independent System Operator), RTO (Regional Transmission Organization), ITP (Independent Transmission Provider), STD (Standard Market Design), and WPMP (Wholesale Power Market Platform) was proposed 2004. All those changes aim to provide true open access to GENCOs, while maintaining the reliability of electric power supply. Although an all-around universal market framework may not exit, lessons have been learned

that more structural changes and coordination need to be done. The blackout of Aug 14th, 2003 which occurred at the edges of PJM and MWISO seemed to support FERC's vision that a free-traded market yet well-coordinated electric power system. Besides market frameworks, other regulations changes are evolving parallel with re-regulation of the electric power industry, such as policies regarding renewable energy and environments. At the same time, repeal of PUHCA and PURPA is under discussion in the legislative sector of US. Beside legislation evolution, electricity markets also see structural changes. The time-varying and location-dependent value of electricity is recognized. Location Marginal Prices framework is implemented by PJM, MISO and other regional electricity markets. ERCOT is transiting from zonal prices to nodal prices, while the California electricity market redesign also adopts nodal prices.

A.2 A Brief Comparison of Some Exiting Electricity Markets

The market structures adopted in exiting electricity markets are, to a large extent, characterized by the scope of activities and authority delegated to the entity responsible for day-to-day operation of the transmission system, i.e., the system operator. These features vary widely among the different ISOs existing or emerging in the U.S. and other countries. To facilitate the comparison of market structures, it is helpful to consider the ISO's role and responsibilities in each of the following areas:

- Operations planning and scheduling
- Dispatching of generation resources
- Real-time transmission system control and monitoring
- On-line network security analysis
- Market operations and settlements
- Transmission planning, ownership and maintenance

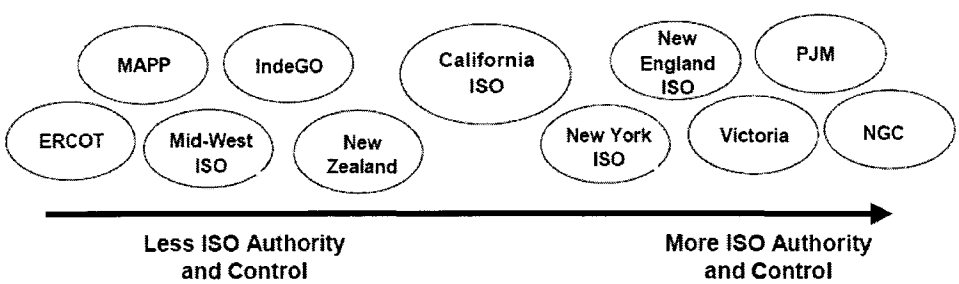
The minimum or core responsibility of all existing and emerging ISOs is the coordination of operations planning within the ISO's area of jurisdiction. A "minimalist ISO" would intervene in operations planning and scheduling only in case the schedules developed by the participants (PX, SCs, etc.) are likely to result in transmission congestion. The ISO would then coordinate measures to alleviate the congestion. The minimalist ISO would not usually perform real-time control through automatic generation control (AGC). It may, however, monitor the operation of the power system to ensure adequacy of available reserves and other pertinent ancillary services. Examples of a minimalist ISO are ERCOT, and the structure being contemplated for MAPP. At the other end of the spectrum, some existing or emerging ISOs have a wide range of authority and enjoy extensive centralized control. In addition to the basic functions of a minimalist ISO, a maximalist ISO would:

- Perform generation scheduling (possibly including unit commitment), and scheduling of ancillary services
- Dispatch generation for energy imbalance and ancillary services, as well as congestion management

- Perform real-time control of generation, transmission, and ancillary resources
- Facilitate a forward (day-ahead and/or hour-ahead) energy market
- Plan and execute transmission system expansion (although it may or may not own the transmission assets).

The PJM ISO is an example of a maximalist ISO. The National Grid Company (NGC) in the U.K. is another example, in which the ISO assumes ownership of transmission assets also.

The following figure shows schematically how the functionalities of different ISO based on the scope and extent of authority and control. In this figure, the degree of ISO authority and control increases from left to right: to the left are those ISOs with minimal authority and control; to the right are those ISOs with maximal authority and centralized control.



A.3 Time Sequence of Key Activities in CAISO and PX Market Operations

Day Ahead	California ISO	California PX
2 days ahead – by 6 PM	Evaluate & publish public market information.	
Day ahead – 6:00 to 6:30 AM	Receive SC load forecasts; aggregate DAC loads; send aggregated DAC loads to UDCs.	
By 7:00 AM		Receive participants' portfolio energy supply & demand bids for each hour.
By 7:15 AM		Conduct energy auction & notify successful bidders of hourly MCPs & quantities.
By 9:10 AM		Receive participants' Initial Preferred Schedules, identifying specific generating units & loads that fulfill aggregate awards in the energy auction; receive adjustment bids for inter-zonal congestion management.
By 9:30 AM		Receive A/S bids.
By 10:00 AM	Receive & validate preferred energy & self-provided A/S schedules & bids from all SCs.	Submit to ISO preferred energy schedules, A/S & adjustment bids.
10:00 to 11:00 AM	Perform A/S auction & inter-zonal congestion management; develop & publish adjusted energy schedules, A/S schedules & MCPs, & estimated congestion charges. NOTE: energy & A/S schedules will be Final if there is no inter-zonal congestion.	
By 12:00 noon	If 10 AM schedules had inter-zonal congestion, receive & validate revised preferred energy & self-provided A/S schedules & bids.	Submit to ISO revised schedules. PX always submits same energy schedules as 10 AM, but may have revised A/S bids.
12:00 noon to 1:00 PM	Perform A/S auction & inter-zonal congestion management; develop & publish Final energy schedules, A/S schedules & MCPs, & congestion charges.	
By 1:15 PM		Send to participants Final energy & A/S schedules & congestion charges; calculate zonal MCPs.
By 1:30 PM approx.	Determine any deficiencies in A/S markets; evaluate RMR requirements relative to Final schedules.	
By 5:00 PM approx.	Publish any changes to Final schedules due to A/S shortfall & RMR requirements.	

Hour Ahead	California ISO	California PX
By 3 hours ahead		Receive participants' energy supply & demand bids, relative to Final DA schedules.
By 2 hrs 50 min ahead		Calculate MCPs & quantities, determine preferred schedules.
By 2 hours ahead	Receive & validate energy schedules, & self-provided A/S schedules & bids.	Receive participants' adjustment & A/S bids; include with preferred schedules submitted to ISO.
2 hrs to 1 hr ahead	Perform A/S auction & congestion management; develop & publish Final energy schedules, A/S schedules & MCPs, congestion charges, & GMMs.	
By 1 hour ahead		Transmit ISO Final schedules to participants.
Prior to operating hour		Calculate & publish zonal MCPs.
Real-time – Prior to Operating Hour	**California ISO**	**California PX**
By 1 hour ahead		Receive participants' supplemental energy bids.
By 45 min ahead of operating hour	Receive supplemental energy bids for real-time market.	
By 20 min ahead	Accept ETC schedules not already scheduled in DA or HA markets.	
Real-time – Within Operating Hour	**California ISO**	**California PX**
By 10 minutes ahead of operating instant	Receive actual system load & MW generation on AGC (from PMS).	
10 min. ahead to operating instant	Determine energy imbalances & dispatch winning supplemental bids via PMS.	

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